Designing of supplementary controller for STATCOM for mitigation of sub-synchronous resonance in series compensated power system

Vipin Jain* and Narendra Kumar**

In FACTS devices supplementary signals are widely used to enhance damping and mitigation of Subsynchronous Resonance in Power System. Subsynchronous Resonance occurs due to series capacitor in the Power Systems. High value of series capacitive reactance may destabilize low frequency mode which is more dangerous. In this paper modeling of STATCOM with IEEE first benchmark model is presented. Then a supplementary signal is developed which is capable to make the system stable for all critical values of series compensation. The eigenvalues are presented for all four critical values of series compensation.

Keywords: STATCOM, supplementary signals, power system stability, IEEE first benchmark model.

1.0 INTRODUCTION

A Static Synchronous Compensator (STATCOM) is also known as an advanced static VAR compensator. It is capable of generating or absorbing reactive power [1]. STATCOM is a shunt connected device and used to control the transmission line voltage but when supplementary signal is used as a feedback signal then it can enhance the damping of the system. In STATCOM type '1' or type '2' converters can be used. In type '1' converters both K_{cs} and ' α ' are controlled and a DC battery is provided in parallel with capacitor. Magnitude of E_s and voltage across DC side capacitor is controlled by controlling K_{cs} therefore reactive power is controlled through K_{cs}. Active power of STATCOM is controlled through ' α '. In type '2' converters only ' α ' can be controlled and 'K_{cs}' is kept fixed. ' α ' alone can control Magnitude and angle of E_s, voltage across DC side capacitor, active power consumed by the STATCOM and reactive power consumed (or supplied) by the STATCOM. For 'p' pulse converter $K_{cs} = (p/6)$ $(\sqrt{6\pi})$ [2]. For 12 pulse converter K_{cs}=2($\sqrt{6\pi}$).

' α ' is the angle difference between E_t and E_s . ' α ' is kept very small (maximum value of ' α ' is kept $\pm 1^\circ$). Low value of ' α ' can control desired reactive power and voltage of the system. For low value of ' α ', Reactive power supplied by STATCOM have linear relationship with ' α '. STATCOM consume small amount of active power [3-6]. As the ' α ' varies, V_{dc} (Voltage across DC side capacitor) varies, then magnitude of 'E_s' varies. |Es| (=K_{cs}V_{dc}), which controls reactive power supplied (or absorbed) by the STATCOM.

2.0 STUDY SYSTEM

Study system considered is IEEE First Benchmark Model [7]. IEEE First Benchmark Model has twenty differential equations. Its generator equivalent circuit has six differential equations, Mechanical system has twelve differential equations, Transmission network (i.e. series capacitor) has two differential equations. STATCOM is incorporated at Generator bus as shown in Figure 1. X_{Ts} is the sending end transformer leakage reactance and X_{Tr} is the receiving end transformer leakage reactance.

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STATCOM has three differential equations. Two differential equation of STATCOM are due to currents through STATCOM ($I_{sd} \& I_{sq}$) and one differential equation is due to V_{dc} . Generator and turbine system have six masses. All the equations of First Benchmark Model are

written as equations 1-6 & 9-20. (This contains STATCOM currents Isd & Isq also). Equations 21-23 show STATCOM dynamics. Equation 24 shows PI controller dynamics. All the equations are developed in simulink and kept in different subsystems as shown in Figure 2.



Output of STATCOM is current I_{sd} , I_{sq} , \dot{I}_{sd} and \dot{I}_{sq} . Input to STATCOM is STATCOM bus voltage (or Generator terminal voltage) e_d & e_q. By keeping all the differential equations in a subsystem these can be linearized and subsequently eigenvalues, impulse response, root locus, observability etc. can be obtained. The block diagram of system is shown in Figure 3. The whole system is kept in one subsystem as shown in Figure 4 to linearise the system and obtain the eigenvalues and root locus. Generator circuit is shown in Figures 5 & 6. Equations 1-6 are obtained from Figures 5 & 6. I_{1d} , I_{1q} , I_{2q} are the damper winding currents. I_{f} is field winding current. All the equations 1-26 are written in d-q reference frame. d-q axis is machine reference frame and D-Q axis is network reference frame (Figure 7). D-Q axis is rotating at synchronous speed. E_{ref} is along the 'D' axis. E_{ref} is infinite bus bar. 'A' & 'B' are the output equations of FMB. (This is input to the STATCOM). Output of PI controller is ' α ', which is input to the STATCOM. Equation 'C' is the output equation of PI controller. Angle of bus Es is $\theta_s = \theta_t + \alpha$. θ_t is angle of E_t . In case of disturbance θ_t varies but as an input to θ_s it remain constant due to phase locked loop (PLL). PLL synchronizes GTO pulses to the system voltage and provides

a reference angle to the measurement system. In lineaisation $\Delta \theta_s = \Delta \theta_t + \Delta \alpha$, but $\Delta \theta_t = 0$. Hence $\Delta \theta_s = \Delta \alpha$. Feedback supplementary signal is passed through lead compensation.









3.0 DIFFERENTIAL EQUATION OF THE SYSTEM

3.1 Generator Equations

$$e_{d} = -R_{a}I_{d} - \frac{(X_{1} + X_{md})}{\omega_{0}}\dot{I}_{d} + \frac{X_{md}}{\omega_{0}}\dot{I}_{f} + \frac{X_{md}}{\omega_{0}}\dot{I}_{ld} - \omega\psi_{q}$$

$$\psi_{q} = -(X_{1} + X_{mq})I_{q} + X_{mq}I_{1q} + X_{mq}I_{2q}$$

$$-\frac{(X_{1} + X_{md})}{\omega_{0}}\dot{I}_{d} + \frac{X_{md}}{\omega_{0}}\dot{I}_{f} + \frac{X_{md}}{\omega_{0}}\dot{I}_{ld} = R_{a}I_{d} - \omega(X_{1} + X_{mq})I_{q}$$

$$+\omega X_{mq}I_{1q} + \omega X_{mq}I_{2q} + e_{d} \qquad \dots (1)$$

$$e_{q} = -R_{a}I_{q} - \frac{(X_{1} + X_{mq})}{\omega_{0}}I_{q} + \frac{X_{mq}}{\omega_{0}}I_{1q} + \frac{X_{mq}}{\omega_{0}}I_{2q} + \omega\psi_{d}$$

$$\psi_{d} = -(X_{1}+X_{md})I_{d}+X_{md}I_{f}+X_{md}I_{ld}$$

$$\frac{-(X_1+X_{mq})}{\omega_0} \dot{I}_q + \frac{X_{mq}}{\omega_0} \dot{I}_{1q} + \frac{X_{mq}}{\omega_0} \dot{I}_{2q} = \omega(X_1+X_{md}) I_d$$
$$+R_a I_q - \omega X_{md} I_f - \omega X_{md} I_{1d} + e_q \qquad \dots (2)$$

$$\frac{-X_{md}}{\omega_0}\dot{I}_d + \frac{X_{fd} + X_{md}}{\omega_0}\dot{I}_f + \frac{X_{md}}{\omega_0}\dot{I}_{1d} = -R_{fd}I_f + e_{fd} \qquad \dots (3)$$

$$\frac{-X_{md}}{\omega_0} \dot{I}_d + \frac{X_{md}}{\omega_0} \dot{I}_f + \frac{X_{1d} + X_{md}}{\omega_0} \dot{I}_{1d} = -R_{1d} I_{1d} \qquad \dots (4)$$

$$\frac{-X_{mq}}{\omega_{0}}\dot{I}_{q} + \frac{X_{1q} + X_{mq}}{\omega_{0}}\dot{I}_{1q} + \frac{X_{mq}}{\omega_{0}}\dot{I}_{2q} = -R_{1q}I_{1q} \qquad \dots (5)$$

$$\frac{-X_{mq}}{\omega_0}\dot{I}_q + \frac{X_{mq}}{\omega_0}\dot{I}_{1q} + \frac{X_{2q} + X_{mq}}{\omega_0}\dot{I}_{2q} = -R_{2q}I_{2q} \dots (6)$$

3.2 Mechanical System Equations

Exciter equations are :

$$2H_{E} \omega_{E_{A}} = K_{GE} (\delta - \delta_{E}) - D_{E} \omega_{E_{A}} \qquad \dots (7)$$
$$\frac{1}{\omega_{0}} \dot{\delta}_{E} = \omega_{E_{A}} \qquad \dots (8)$$

 ω_{E_A} is accelerating frequency of the exciter mode. $\omega_{E_A} = \omega_E - \omega_S$ where ω_E is the angular speed of exciter mode. ω_S is the synchronous speed of rotating reference frame. In steady state $\omega_E = \omega_S = 1$ pu. In Transient period ω_E may be other than 1, but $\omega_S = 1$. Hence $\dot{\omega}_s = 0$.

Therefore equations 7 & 8 can be written as

$$2H_{E} \omega_{E} = K_{GE} (\delta - \delta_{E}) - D_{E} (\omega_{E} - \omega_{S}) \qquad \dots (9)$$

$$\frac{1}{\omega_0} \dot{\delta}_E = \omega_E - \omega_S \qquad \dots (10)$$

similarly generator rotor mode equations are

$$2H_{G}\omega = K_{BG}(\delta_{B} - \delta) - T_{e} - K_{GE}(\delta - \delta_{E}) - D_{G}(\omega - \omega_{S}) \qquad \dots (11)$$

$$\frac{1}{\omega_0}\dot{\delta} = \omega - \omega_s \qquad \dots (12)$$

where, T_e is electrical torque generated $T_e = \psi_d I_q - \psi_q I_d$ $T_e = -(X_1 + X_{md})I_d I_q + X_{md}I_f I_q + X_{md}I_{1d}I_q$ $+(X_1 + X_{mq})I_q I_d - X_{mq}I_{1q}I_d - X_{mq}I_{2q}I_d$

LP-B turbine equations are

$$2H_{B} \omega_{B} = T_{LPB} + K_{AB} (\delta_{A} - \delta_{B}) - K_{BG} (\delta_{B} - \delta)$$
$$-D_{B} (\omega_{B} - \omega_{S}) \qquad \dots (13)$$

$$\frac{1}{\omega_0} \dot{\delta}_{\rm B} = \omega_{\rm B} - \omega_{\rm S} \qquad \dots (14)$$

LP-A turbine equations are

$$2H_{A} \omega_{A} = T_{LPA} + K_{IA} (\delta_{I} - \delta_{A}) - K_{AB} (\delta_{A} - \delta_{B}) - D_{A} (\omega_{A} - \omega_{S}) \qquad \dots (15)$$

$$\frac{1}{\omega_0} \dot{\delta}_A = \omega_A - \omega_S \qquad \dots (16)$$

IP turbine equations are

$$\frac{1}{\omega_0} \dot{\delta}_1 = \omega_1 - \omega_s \qquad \dots (18)$$

HP turbine equations are

$$2H_{\rm H} \omega_{\rm H} = T_{\rm HP} - K_{\rm HI} (\delta_{\rm H} - \delta_{\rm I})$$
$$-D_{\rm H} (\omega_{\rm H} - \omega_{\rm S}) \qquad \dots (19)$$

$$\frac{1}{\omega_0} \dot{\delta}_{\rm H} = \omega_{\rm H} - \omega_{\rm S} \qquad \dots (20)$$

3.3 Transmission Line Equations

$$\begin{split} & e_{d} - E_{ref} \sin \delta - V_{cd} = R_{L} \left(I_{d} + I_{sd} \right) - \omega X_{TL} \left(I_{q} + I_{sq} \right) \\ & + \frac{X_{TL}}{\omega_{0}} \left(\dot{I}_{d} + \dot{I}_{sd} \right) \end{split}$$

or,
$$e_d = E_{ref} \sin \delta + V_{cd} + R_L (I_d + I_{sd})$$

 $-\omega X_{TL} (I_q + I_{sq}) + \frac{X_{TL}}{\omega_0} (I_d + I_{sd})$
 $e_q - E_{ref} \cos \delta - V_{cq} = R_L (I_q + I_{sq}) + \omega X_{TL} (I_d + I_{sd})$
 $+ \frac{X_{TL}}{\omega_0} (I_q + I_{sq})$
or, $e_q = E_{ref} \cos \delta + V_{cq} + R_L (I_q + I_{sq}) + \omega X_{TL} (I_d + I_{sd})$
 $+ \frac{X_{TL}}{\omega_0} (I_q + I_{sq})$ (B)

$$\frac{V_{cd}}{\omega_0} = \omega V_{cq} + X_c (I_d + I_{sd}) \qquad \dots (21)$$

$$\frac{V_{cq}}{\omega_0} = -\omega V_{cd} + X_c (I_q + I_{sq}) \qquad \dots (22)$$

where X_{TL} is the effective reactance of Transmission line i.e. $X_{TL} = X_{TS} + X_L + X_{Tr}$

(V_{cd} and V_{cq} are the voltage across series capacitor)

3.4 Statcom Circuit Equations

$$E_{sd} - e_d = R_s I_{sd} - \omega X_s I_{sq} + \frac{X_s I_{sd}}{\omega_0} \qquad \dots (23)$$

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$$E_{sq} - e_{q} = R_{s}I_{sq} + \omega X_{s}I_{sd} + \frac{X_{s}I_{sq}}{\omega_{0}} \qquad \dots (24)$$

$$E_{sd} = K_{cs} V_{dc} \cos\theta_{s}, E_{sq} = K_{cs} V_{dc} \sin\theta_{s}$$

Linearization of $E_{sd} \& E_{sq}$ is given here:
$$\Delta E_{sd} = K_{cs} \cdot \Delta V_{dc} \cdot \cos\theta_{s0} + K_{cs} V_{dc0} (-\sin\theta_{s0}) \Delta \alpha$$
$$\Delta E_{sq} = K_{cs} \cdot \Delta V_{dc} \cdot \sin\theta_{s0} + K_{cs} V_{dc0} \cos\theta_{s0} \Delta \alpha$$

STATCOM DC side Power and AC side active power at node E_s must be equal. In steady state:

$$V_{dc}I_{dc} = \text{Real}\left[\left(E_{sd} + jE_{sq}\right)\left(I_{sd} + jI_{sq}\right)^{*}\right] = E_{sd}I_{sd}$$
$$+E_{sq}I_{sq}$$
or, $V_{dc}I_{dc} = K_{cs}V_{dc}\cos\theta_{s}I_{sd} + K_{cs}V_{dc}\sin\theta_{s}I_{sq}$ or, $I_{dc} = K_{cs}\cos\theta_{s}I_{sd} + K_{cs}\sin\theta_{s}I_{sq}$

This is the expression of DC side current. In steady state numerical value of I_{dc} should be negative because R_p is consuming DC power, while I_{dc} shown in Figure 1 is in opposite direction. Current through DC side capacitor

$$= C_{s} \dot{V}_{dc} = -(I_{dc} + \frac{V_{dc}}{R_{p}})$$

or, $\dot{V}_{dc} = -\frac{1}{C_{s}} (\frac{V_{dc}}{R_{p}} + K_{cs} I_{sd} \cos\theta_{s}$
 $+ K_{cs} I_{sq} \sin\theta_{s})$ (25)

3.5 PI Controller Equation

$$\dot{x} = V_{aux} + V_{ref} - |E_t| + K_d I_{sq}$$
(26)

Output equation of PI controller is:

$$\alpha = K_i x + K_p x \qquad \dots (C)$$

All the differential equations are developed in simulink and linearized in simulink.

4.0 SUPPLEMENTARY CONTROLLER DESIGN

To design a supplementary controller whole system is kept in the subsystem as shown in Figure 4. The system is termed as 'sys1'. Output of system may be any state variable or a quantity (eigenvalues do not depend on output). Various supplementary signals are tested such as deviation of speed of generator rotor mode (w- ω_s), deviation in reactive power (ΔQ), deviation in active power (ΔP), deviation in generator current magnitude (ΔI), deviation in STATCOM current magnitude (ΔI_s) , deviation in Statcom DC side capacitor current ($C_s V_{dc}$). It is found that ω - ω_s is most suitable supplementary signal. It is shown in Table 1-4 that with this supplementary signal all eigenvalues are in the left Half of s-plane. Table 1 shows the eigenvalues when $X_c = 0.185$. This critical value of X_c makes the Torsional mode 4 most unstable, because SSR frequency (subsynchronous frequency) is also in the range of 202. When supplementary signal ω - ω_s is incorporated then all eigenvalues are shifted to left half of s-plane.Similarly Table 2 shows the

eigenvalues when $X_c = 0.284$. This critical value of X_c makes the Torsional mode 3 most unstable, because SSR frequency is also in the range of 160. In this case also supplementary signal makes the system stable. Table 3 shows the eigenvalues when $X_c = 0.379$. This critical value of X_c tends the Torsional mode 2 most unstable, because SSR frequency is also in the range of 126. In this case also supplementary signal makes the system stable. Table 4 shows the eigenvalues when $X_c = 0.473$. This critical value of X_c makes the Torsional mode 1 most unstable, because SSR frequency is also in the range of 98. In this case also supplementary signal makes the system stable.

Time response for various quantities and shaft torque are shown in Figures 8-15. Time response is shown against $X_c = 0.473$.

5.0 CONCLUSION

From the above analysis it can be concluded that supplementary signal deviation in speed of generator rotor mode is a very suitable supplementary signal which can make the system against all four critical values of X_c . In our research we have also found that when Natural damping is kept equal to one for all six modes then without supplementary signal system is unstable against aforesaid four critical values of X_c but system is stable when supplementary signal is incorporated.

















6.0 APPENDICES

Appendix A

- Initial Condition of the system at $X_c=0.473$ (all quantities are in pu, unless otherwise specified). Generator circuit data: $R_a=0$, $X_l=0.13$, $X_{md}=1.66$, $R_{1d}=0.00408$, $X_{1d}=0.0055$, $R_{fd}=0.001406$, $X_{fd}=0.062$, $X_{mq}=1.58$, $R_{1q}=0.00822$, $X_{1q}=0.095$, $R_{2q}=0.01406$, $X_{2q}=0.326$.
- Power supplied by generator $P_g = 0.9$, PF= 0.9(lagging). Current supplied by Generator $I_G=0.914+j0.387$. Terminal Voltage of Generator is E_t .

Where $E_t = |E_t| \angle \theta_t = e_d + je_q = 1.0089 \angle 49.04^\circ$ In steady state, damper winding currents are zero (i.e. $I_{1d} = I_{2d} = I_{2q} = 0$). $\psi_q = -0.6613$, $\psi_d = 0.762$. $I_f = 1.445$.

• I_s=-0.1717+j0.1442. I_s is the current supplied by Statcom. Statcom is consuming active power to meet out losses in R_s and R_p. STATCOM is consuming Reactive power, because $|E_t| = 1.0089 \& |E_s| = 0.975$, Therefore STATCOM is in inductive mode. Infinite bus is $E_{ref} = 0.963$ (along the 'D' axis). But in d-q reference frame it is $0.963 \angle 37.23^{\circ}$ i.e. $\delta = 52.77^{\circ}$ (Figure 7). STATCOM circuit data : $R_s = 0.01$, $X_s=0.15$, $R_p=125$, $C_s=0.015$, $\alpha=0.1^\circ$, $\theta_s=\theta_t+\alpha$. $K_{cs}=2\sqrt{6}/\pi$, $|E_s|=K_{cs}V_{dc}=0.975$, $V_{dc}=0.625$, Active Power consumed by STATCOM (i.e R_s and R_p)=0.00363, Active Power consumed by $R_s = 0.0005028$, Active Power consumed in switching losses (i.e. in R_p) = 0.003129. In steady state current through capacitor $C_{s'}$ (DC side capacitor of converter) is zero. In steady state Converter consume current V_{dc} equal to R_p , which is to meet out the losses of converter. R_p represents switching losses. I_{dc} = -0.005003. PI controller data: K_p = -1, $K_i = -3.75 \text{ sec}^{-1}$. $K_d = 0.001$, Supplementary controller data: K=15, z=20 sec⁻¹, p=100

• In steady State $T_e = T_m = T_{HP} + T_{IP} + T_{LPA} + T_{LPB} = 0.9$ (T_m is mechanical Torque applied).

sec⁻¹.

 $T_{HP} = 0.3 T_{M}$ $T_{IP}=0.26T_{M}$ $T_{LPA} = 0.22 T_{M},$ $T_{LPB}=0.22T_{M}, \delta_{H}=0.975, \delta_{I}=0.962, \delta_{A}=0.947,$ $\delta_{\rm B}$ =0.934, δ = $\delta_{\rm E}$ =0.921 radian. Torque between HP-IP turbine= $0.3T_m = K_{HI}(\delta_{H})$ δ_{I})=0.270, Torque between IP- LP-A turbine=0.56T_m = $K_{IA}(\delta_I - \delta_A)=0.504$, Torque between LP-A- LP-B turbine=0.78T_m= $K_{AB}(\delta_A - \delta_B) = 0.701$, Torque between LP-Generator= T_m = $K_{BG}(\delta_B$ -B- δ) =0.9, Torque between Generator- Exciter=0. Natural Damping is zero i.e. $D_E = D_G = D_B =$ $D_A = D_I = D_H = 0$. The mechanical system with steady state conditions is shown in Figure 16. Values of $K_{KI} K_{IA}$ etc. and H_E , H_G etc. are given in reference [7].

 $\omega_0 = 2\pi f = 377 \text{rad/sec.}$ In equations 1-26 all values are in per unit except ω_0 , K_i and lead compensation data. Unit of x is seconds and \dot{x} is pu. In lead compensation pole and zero have the unit sec.⁻¹. Unit of Inertia constants H_E, H_G, H_B, H_A, H_I, H_H are seconds.



Appendix B

 $\omega_{\rm E}$, ω , $\omega_{\rm B}$, $\omega_{\rm A}$, $\omega_{\rm I}$, $\omega_{\rm H}$ are the speed of all masses. In steady state $\omega_{\rm E} = \omega = \omega_{\rm B} = \omega_{\rm A} = \omega_{\rm I} = \omega_{\rm H} = \omega_{\rm s} = 1$ pu. In Transient period $\omega_{\rm s} = 1$ pu, but the value of $\omega_{\rm E}$, ω , $\omega_{\rm B}$, $\omega_{\rm A}$, $\omega_{\rm I}$, $\omega_{\rm H}$ may be different from 1 pu. ' ω ' is the generator mode frequency and the frequency at which d-q axis is rotating. ' $\omega_{\rm s}$ ' is the frequency at which D-Q axis is rotating.

Linerarization of two equations 9 & 10 are given here for the convinience of the readers,

$$2H_{E}\Delta\omega_{E} = K_{GE}(\Delta\delta - \Delta\delta_{E}) - D_{E}\Delta\omega_{E} - (L9)$$
$$\frac{1}{\omega_{0}}\Delta\dot{\delta}_{E} = \Delta\omega_{E} - (L10)(\Delta\omega_{S} = 0)$$

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TABLE 1						
EIGENVALUES WITH SUPPLEMENTARY SIGNAL ω - ω_s WITH A LEAD COMPENSATION						
$(AT X_{C}=0.185), P_{G}=0.9, PF=0.9 (LAGGING)$						
Description	Eigenvalues of First Benchmark Model without any controller	Eigenvalues with STATCOM	Eigenvalues with STATCOM and SupplementaryController			
Supersynchronous	-4.6421±551.15i	-5.2555±550.47i	-5.5587±550.17i			
Torsional Mode No. 5	-1.3799e-007±298.18i	-3.7351e-06±298.18i	-6.9351e-06±298.18i			
Torsional Mode No. 4	1.6445±202.81i	0.76248±202.74i	-5.2892±200.79i			
Torsional Mode No. 3	0.013438±160.74i	0.056129±160.72i	-0.24877±160.37i			
Torsional Mode No. 2	0.00094107±127.04i	0.0068329±127.04i	-0.044871±126.99i			
Torsional Mode No. 1	0.00048159± 99.317i	0.048232±99.258i	-0.40271±98.93i			
Subsynchronous	-5.2438±202.58i	-11.296±202.01i	-4.8678±206.25i			
Electromechanical mode	-0.43436±9.4829i	-0.36163±9.0812i	-0.45687±9.0046i			
Statcom currents		-129.73±2286.6i	-130.29±2286.3i			
V_{dc}		-223.69	-217.92			
Lead Compensation			-102.76			
others	-32.498	-32.214	-32.357			
	-20.34	-21.721	-21.714			
	-3.3559	-9.871	-9.9431			
	-0.25653	-1.0991	-1.068			
		-1.4044	-1.4396			
TABLE 2						
	TAB	LE 2				
EIGENVALUES W	TAB ITH SUPPLEMENTARY SI (AT Xc=0.284) Pc=0.9	LE 2 GNAL ω-ω _s WITH A LEAI) PF=0.9 (LAGGING)	D COMPENSATION			
EIGENVALUES W	TAB ITH SUPPLEMENTARY SI (AT X _C =0.284), P _G =0.9 Eigenvalues of First	LE 2 GNAL ω-ω _s WITH A LEAI 9, PF=0.9 (LAGGING)	D COMPENSATION			
EIGENVALUES W	TAB ITH SUPPLEMENTARY SI (AT X _C =0.284), P _G =0.9 Eigenvalues of First Benchmark Model without any controller	LE 2 GNAL ω-ω _s WITH A LEAI 9, PF=0.9 (LAGGING) Eigenvalues with STATCOM	D COMPENSATION Eigenvalues with STATCOM and SupplementaryController			
EIGENVALUES W Description Supersynchronous	TAB TH SUPPLEMENTARY SI (AT X _C =0.284), P _G =0.9 Eigenvalues of First Benchmark Model without any controller -4.6867±592.8i	LE 2 GNAL ω-ω _s WITH A LEAI D, PF=0.9 (LAGGING) Eigenvalues with STATCOM -6.0257±591.98i	COMPENSATION Eigenvalues with STATCOM and SupplementaryController -6.376±591.66i			
EIGENVALUES W Description Supersynchronous Torsional Mode No. 5	TABTABITH SUPPLEMENTARY SIG(AT X_c =0.284), P_G =0.9Eigenvalues of FirstBenchmark Model withoutany controller-4.6867±592.8i-3.9346e-07±298.18i	LE 2 GNAL ω-ω _s WITH A LEAI D, PF=0.9 (LAGGING) Eigenvalues with STATCOM -6.0257±591.98i -4.1297e-06±298.18i	COMPENSATION Eigenvalues with STATCOM and SupplementaryController -6.376±591.66i -5.6414e-06±298.18i			
EIGENVALUES W Description Supersynchronous Torsional Mode No. 5 Torsional Mode No. 4	TAB TAB ITH SUPPLEMENTARY SI (AT X_C =0.284), P_G =0.9 Eigenvalues of First Benchmark Model without any controller -4.6867±592.8i -3.9346e-07±298.18i 0.011845±202.72i	LE 2 GNAL ω-ω _s WITH A LEAI 0, PF=0.9 (LAGGING) Eigenvalues with STATCOM -6.0257±591.98i -4.1297e-06±298.18i 0.032417±202.7i	COMPENSATION Eigenvalues with STATCOM and SupplementaryController -6.376±591.66i -5.6414e-06±298.18i -0.4541±204.19i			
EIGENVALUES W Description Supersynchronous Torsional Mode No. 5 Torsional Mode No. 4 Torsional Mode No. 3	TAB TAB ITH SUPPLEMENTARY SIG (AT X_c =0.284), P_G =0.9 Eigenvalues of First Benchmark Model without any controller -4.6867±592.8i -3.9346e-07±298.18i 0.011845±202.72i 1.5491±160.65i	LE 2 GNAL ω-ω _s WITH A LEAI 0, PF=0.9 (LAGGING) Eigenvalues with STATCOM -6.0257±591.98i -4.1297e-06±298.18i 0.032417±202.7i 0.50523±160.45i	D COMPENSATION Eigenvalues with STATCOM and SupplementaryController -6.376±591.66i -5.6414e-06±298.18i -0.4541±204.19i -2.0409±160.58i			
EIGENVALUES W Description Supersynchronous Torsional Mode No. 5 Torsional Mode No. 4 Torsional Mode No. 3 Torsional Mode No. 2	TAB TAB ITH SUPPLEMENTARY SI- (AT X_c =0.284), P_G =0.9 Eigenvalues of First Benchmark Model without any controller -4.6867±592.8i -3.9346e-07±298.18i 0.011845±202.72i 1.5491±160.65i 0.006224±127.09i	LE 2 GNAL ω-ω _s WITH A LEAI 0, PF=0.9 (LAGGING) Eigenvalues with STATCOM -6.0257±591.98i -4.1297e-06±298.18i 0.032417±202.7i 0.50523±160.45i 0.033788 ±127.07i	D COMPENSATION Eigenvalues with STATCOM and SupplementaryController -6.376±591.66i -5.6414e-06±298.18i -0.4541±204.19i -2.0409±160.58i -0.054356±126.91i			
EIGENVALUES W Description Supersynchronous Torsional Mode No. 5 Torsional Mode No. 4 Torsional Mode No. 3 Torsional Mode No. 2 Torsional Mode No. 1	TAB TAB ITH SUPPLEMENTARY SI- (AT X_c =0.284), P_G =0.9 Eigenvalues of First Benchmark Model without any controller -4.6867±592.8i -3.9346e-07±298.18i 0.011845±202.72i 1.5491±160.65i 0.006224±127.09i 0.013663±99.573i	LE 2 GNAL $ω-ω_8$ WITH A LEAI 0, PF=0.9 (LAGGING) Eigenvalues with STATCOM -6.0257±591.98i -4.1297e-06±298.18i 0.032417±202.7i 0.50523±160.45i 0.033788 ±127.07i 0.14009±99.498i	Eigenvalues with STATCOM and SupplementaryController -6.376±591.66i -5.6414e-06±298.18i -0.4541±204.19i -2.0409±160.58i -0.054356±126.91i -0.26428±98.681i			
EIGENVALUES W Description Supersynchronous Torsional Mode No. 5 Torsional Mode No. 4 Torsional Mode No. 3 Torsional Mode No. 2 Torsional Mode No. 1 Subsynchronous	TABTABITH SUPPLEMENTARY SI (AT X_c =0.284), P_G =0.9Eigenvalues of FirstBenchmark Model without any controller-4.6867±592.8i-3.9346e-07±298.18i0.011845±202.72i1.5491±160.65i0.006224±127.09i0.013663±99.573i-4.7406±160.8i	LE 2 GNAL ω - ω_{s} WITH A LEAI D, PF=0.9 (LAGGING) Eigenvalues with STATCOM -6.0257±591.98i -4.1297e-06±298.18i 0.032417±202.7i 0.50523±160.45i 0.033788 ±127.07i 0.14009±99.498i -14.5±158.7i	COMPENSATION Eigenvalues with STATCOM and SupplementaryController -6.376±591.66i -5.6414e-06±298.18i -0.4541±204.19i -2.0409±160.58i -0.054356±126.91i -0.26428±98.681i -11.277±160.53i			
EIGENVALUES W Description Supersynchronous Torsional Mode No. 5 Torsional Mode No. 3 Torsional Mode No. 2 Torsional Mode No. 1 Subsynchronous Electromechanical mode	TABTABITH SUPPLEMENTARY SI (AT X_C =0.284), P_G =0.9Eigenvalues of First Benchmark Model without any controller-4.6867±592.8i-3.9346e-07±298.18i0.011845±202.72i1.5491±160.65i0.006224±127.09i0.013663±99.573i-4.7406±160.8i-0.54831±10.385i	LE 2 GNAL $ω-ω_8$ WITH A LEAI 0, PF=0.9 (LAGGING) Eigenvalues with STATCOM -6.0257±591.98i -4.1297e-06±298.18i 0.032417±202.7i 0.50523±160.45i 0.033788 ±127.07i 0.14009±99.498i -14.5±158.7i -0.41946±9.9204i	Eigenvalues with STATCOM and SupplementaryController -6.376±591.66i -5.6414e-06±298.18i -0.4541±204.19i -2.0409±160.58i -0.054356±126.91i -0.26428±98.681i -11.277±160.53i -0.50479±9.8282i			
EIGENVALUES W Description Supersynchronous Torsional Mode No. 5 Torsional Mode No. 4 Torsional Mode No. 3 Torsional Mode No. 2 Torsional Mode No. 1 Subsynchronous Electromechanical mode Statcom currents	TABTABITH SUPPLEMENTARY SI (AT X_c =0.284), P_G=0.9Eigenvalues of FirstBenchmark Model without any controller-4.6867±592.8i-3.9346e-07±298.18i0.011845±202.72i1.5491±160.65i0.006224±127.09i0.013663±99.573i-4.7406±160.8i-0.54831±10.385i	LE 2 GNAL ω - ω_{s} WITH A LEAI D, PF=0.9 (LAGGING) Eigenvalues with STATCOM -6.0257±591.98i -4.1297e-06±298.18i 0.032417±202.7i 0.50523±160.45i 0.033788 ±127.07i 0.14009±99.498i -14.5±158.7i -0.41946±9.9204i -129.72±2286.7i	Eigenvalues with STATCOM and SupplementaryController -6.376±591.66i -5.6414e-06±298.18i -0.4541±204.19i -2.0409±160.58i -0.054356±126.91i -0.26428±98.681i -11.277±160.53i -0.50479±9.8282i -130.29±2286.5i			
EIGENVALUES W Description Supersynchronous Torsional Mode No. 5 Torsional Mode No. 4 Torsional Mode No. 3 Torsional Mode No. 2 Torsional Mode No. 1 Subsynchronous Electromechanical mode Statcom currents V _{dc}	TAB TAB ITH SUPPLEMENTARY SI (AT X_c =0.284), P_G =0.9 Eigenvalues of First Benchmark Model without any controller -4.6867±592.8i -3.9346e-07±298.18i 0.011845±202.72i 1.5491±160.65i 0.006224±127.09i 0.013663±99.573i -4.7406±160.8i -0.54831±10.385i	LE 2 GNAL $ω-ω_8$ WITH A LEAI D, PF=0.9 (LAGGING) Eigenvalues with STATCOM -6.0257±591.98i -4.1297e-06±298.18i 0.032417±202.7i 0.50523±160.45i 0.033788±127.07i 0.14009±99.498i -14.5±158.7i -0.41946±9.9204i -129.72±2286.7i -215.67	Eigenvalues with STATCOM and SupplementaryController -6.376±591.66i -5.6414e-06±298.18i -0.4541±204.19i -2.0409±160.58i -0.054356±126.91i -0.26428±98.681i -11.277±160.53i -0.50479±9.8282i -130.29±2286.5i -211.53			
EIGENVALUES W Description Supersynchronous Torsional Mode No. 5 Torsional Mode No. 3 Torsional Mode No. 3 Torsional Mode No. 2 Torsional Mode No. 1 Subsynchronous Electromechanical mode Statcom currents V _{dc} Lead Compensation	TAB ITH SUPPLEMENTARY SI- (AT X_c =0.284), P_G=0.9 Eigenvalues of First Benchmark Model without any controller -4.6867±592.8i -3.9346e-07±298.18i 0.011845±202.72i 1.5491±160.65i 0.006224±127.09i 0.013663±99.573i -4.7406±160.8i -0.54831±10.385i	LE 2 GNAL $\omega - \omega_{s}$ WITH A LEAI D, PF=0.9 (LAGGING) Eigenvalues with STATCOM -6.0257±591.98i -4.1297e-06±298.18i 0.032417±202.7i 0.50523±160.45i 0.033788 ±127.07i 0.14009±99.498i -14.5±158.7i -0.41946±9.9204i -129.72±2286.7i -215.67	Eigenvalues with STATCOM and SupplementaryController -6.376±591.66i -5.6414e-06±298.18i -0.4541±204.19i -2.0409±160.58i -0.054356±126.91i -0.26428±98.681i -11.277±160.53i -0.50479±9.8282i -130.29±2286.5i -211.53 -101.3			
EIGENVALUES W Description Supersynchronous Torsional Mode No. 5 Torsional Mode No. 4 Torsional Mode No. 3 Torsional Mode No. 2 Torsional Mode No. 2 Torsional Mode No. 1 Subsynchronous Electromechanical mode Statcom currents Vdc Lead Compensation others	TABI TABI ITH SUPPLEMENTARY SI- (AT X_c =0.284), P_G =0.9 Eigenvalues of First Benchmark Model without any controller -4.6867±592.8i -3.9346e-07±298.18i 0.011845±202.72i 1.5491±160.65i 0.006224±127.09i 0.013663±99.573i -4.7406±160.8i -0.54831±10.385i -32.724	LE 2 GNAL ω - ω_s WITH A LEAI D, PF=0.9 (LAGGING) Eigenvalues with STATCOM -6.0257±591.98i -4.1297e-06±298.18i 0.032417±202.7i 0.50523±160.45i 0.033788±127.07i 0.14009±99.498i -14.5±158.7i -0.41946±9.9204i -129.72±2286.7i -215.67 -32.343	Eigenvalues with STATCOM and SupplementaryController -6.376±591.66i -5.6414e-06±298.18i -0.4541±204.19i -2.0409±160.58i -0.054356±126.91i -0.26428±98.681i -11.277±160.53i -0.50479±9.8282i -130.29±2286.5i -211.53 -101.3 -32.499			
EIGENVALUES W Description Supersynchronous Torsional Mode No. 5 Torsional Mode No. 3 Torsional Mode No. 3 Torsional Mode No. 2 Torsional Mode No. 1 Subsynchronous Electromechanical mode Statcom currents V _{dc} Lead Compensation others	TABITABIITH SUPPLEMENTARY SI (AT X_c =0.284), P_G=0.9Eigenvalues of FirstBenchmark Model without any controller-4.6867±592.8i-4.6867±592.8i-3.9346e-07±298.18i0.011845±202.72i1.5491±160.65i0.006224±127.09i0.013663±99.573i-4.7406±160.8i-0.54831±10.385i-32.724-32.724-20.384	LE 2 GNAL $ω-ω_8$ WITH A LEAI D, PF=0.9 (LAGGING) Eigenvalues with STATCOM -6.0257±591.98i -4.1297e-06±298.18i 0.032417±202.7i 0.50523±160.45i 0.033788 ±127.07i 0.14009±99.498i -14.5±158.7i -0.41946±9.9204i -129.72±2286.7i -215.67 -32.343 -21.709	Eigenvalues with STATCOM and SupplementaryController -6.376±591.66i -5.6414e-06±298.18i -0.4541±204.19i -2.0409±160.58i -0.054356±126.91i -0.26428±98.681i -11.277±160.53i -0.50479±9.8282i -130.29±2286.5i -101.3 -32.499 -21.702			
EIGENVALUES W Description Supersynchronous Torsional Mode No. 5 Torsional Mode No. 3 Torsional Mode No. 2 Torsional Mode No. 2 Torsional Mode No. 1 Subsynchronous Electromechanical mode Statcom currents V _{dc} Lead Compensation others	TABI TABI ITH SUPPLEMENTARY SI- (AT X_c =0.284), P_G =0.9 Eigenvalues of First Benchmark Model without any controller -4.6867±592.8i -3.9346e-07±298.18i 0.011845±202.72i 1.5491±160.65i 0.006224±127.09i 0.013663±99.573i -4.7406±160.8i -0.54831±10.385i -32.724 -20.384 -3.567	LE 2 GNAL ω - ω_s WITH A LEAI D, PF=0.9 (LAGGING) Eigenvalues with STATCOM -6.0257±591.98i -4.1297e-06±298.18i 0.032417±202.7i 0.50523±160.45i 0.033788±127.07i 0.14009±99.498i -14.5±158.7i -0.41946±9.9204i -129.72±2286.7i -215.67 -32.343 -21.709 -9.8483	Eigenvalues with STATCOM and SupplementaryController -6.376±591.66i -5.6414e-06±298.18i -0.4541±204.19i -2.0409±160.58i -0.054356±126.91i -0.26428±98.681i -11.277±160.53i -0.50479±9.8282i -130.29±2286.5i -211.53 -101.3 -32.499 -21.702 -9.9203			
EIGENVALUES W Description Supersynchronous Torsional Mode No. 5 Torsional Mode No. 3 Torsional Mode No. 3 Torsional Mode No. 1 Subsynchronous Electromechanical mode Statcom currents V _{dc} Lead Compensation others	TABI TABI ITH SUPPLEMENTARY SI (AT X_c =0.284), P_G=0.9 Eigenvalues of First Benchmark Model without any controller -4.6867±592.8i -3.9346e-07±298.18i 0.011845±202.72i 1.5491±160.65i 0.006224±127.09i 0.013663±99.573i -4.7406±160.8i -0.54831±10.385i -32.724 -20.384 -3.567 -0.3075	LE 2 GNAL $ω-ω_s$ WITH A LEAI D, PF=0.9 (LAGGING) Eigenvalues with STATCOM -6.0257±591.98i -4.1297e-06±298.18i 0.032417±202.7i 0.50523±160.45i 0.033788 ±127.07i 0.14009±99.498i -14.5±158.7i -0.41946±9.9204i -129.72±2286.7i -215.67 -32.343 -21.709 -9.8483 -1.1193	Eigenvalues with STATCOM and SupplementaryController -6.376±591.66i -5.6414e-06±298.18i -0.4541±204.19i -2.0409±160.58i -0.054356±126.91i -0.26428±98.681i -11.277±160.53i -0.50479±9.8282i -130.29±2286.5i -211.53 -101.3 -32.499 -21.702 -9.9203 -1.0819			

TABLE 3					
EIGENVALUES WITH SUPPLEMENTARY SIGNAL ω - ω_s WITH A LEAD COMPENSATION (AT X =0.379) P =0.9 PE=0.9 (I A GGING)					
Description	Eigenvalues of First Benchmark Model without any controller	Eigenvalues with STATCOM	Eigenvalues with STATCOM and SupplementaryController		
Supersynchronous	-4.7182±626.31i	-6.7181±625.45i	-7.0939±625.14i		
Torsional Mode No. 5	-5.0166e-07±298.18i	-4.3304e-06 ±298.18i	-4.4248e-06±298.18i		
Torsional Mode No. 4	0.00070737±202.81i	0.0048066±202.78i	-0.34714±203.95i		
Torsional Mode No. 3	0.014779±160.36i	0.059281±160.39i	-0.54808±161.33i		
Torsional Mode No. 2	0.76361±126.97i	0.13377±126.96i	-0.38242±126.94i		
Torsional Mode No. 1	0.097558±100.29i	0.72912±99.885i	-0.3244±97.706i		
Subsynchronous	-3.5038±126.82i	-18.148±122.28i	-16.07±125.98i		
Electromechanical mode	-0.70054±11.35i	-0.45221±10.817i	-0.52844±10.7i		
Statcom currents		-128.38±2288.7i	-128.94±2288.5i		
V_{dc}		-205.96	-203.64		
Lead Compensation			-99.155		
others	-33.021	-32.363	-32.534		
	-20.443	-21.728	-21.72		
	-3.8582	-9.8743	-9.9464		
	-0.3579	-1.1611	-1.1081		
		-1.3289	-1.3886		

TABLE 4

EIGENVALUES WITH SUPPLEMENTARY SIGNAL ω - ω_s WITH A LEAD COMPENSATION (AT X_c=0.473), P_g=0.9, PF=0.9 (LAGGING)

Description	Eigenvalues of First Benchmark Model without any controller	Eigenvalues with STATCOM	Eigenvalues with STATCOM and SupplementaryController
Supersynchronous	-4.743±655.53i	-7.3405±654.73i	-7.7269±654.43i
Torsional Mode No. 5	-5.7053e-07±298.18i	-4.4858e-06±298.18i	-2.2407e-06±298.18i
Torsional Mode No. 4	-0.001768±202.85i	-0.0026789±202.82i	-0.31307±203.88i
Torsional Mode No. 3	0.0010519± 160.47i	0.01664±160.46i	-0.38086±161.19i
Torsional Mode No. 2	0.0050084±126.93i	0.024419±126.96i	-0.21189±127.16i
Torsional Mode No. 1	5.0657±98.896i	1.3138±98.446i	-4.0006±98.086i
Subsynchronous	-6.9566±98.739i	-22.252±90.553i	-17.069±93.921i
Electromechanical mode	-0.93128±12.519i	-0.3943±11.892i	-0.45931±11.725i
Statcom currents		-125.93±2291.9i	-126.47±2291.6i
V _{dc}		-194.4	-194.43
Lead Compensation			-95.582
others	-33.44	-32.205	-32.392
	-20.527	-21.763	-21.755
	-4.265	-9.9189	-9.9905
	-0.41811	-1.2362±0.10198i	-1.1866
			-1.2934