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Comparison of Tracing Based Real Power Transmission Loss Allocation Methods in Deregulated Power System

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This paper compares the transmission loss allocation procedures through tracing using proportional sharing rule and provides a detailed comparison of three alternative methods: 1) Graph based tracing 2) Matrix based tracing and 3) Complex power flow tracing. The methods are based on tracing the real and reactive power flow through the network and determining the share of each load on the flow and losses through each line. Power flows of generators and loads are traced to determine the transmission system usage by each generator and load. Then transmission losses caused by each generator or load are determined. Unbundling, (electric energy can be separated commercially as a product from transmission as a service) an idea, which the current deregulation market hinges on is carried out and considers the coupling between active and reactive power flows as well as the cross effects of active and reactive power on active and reactive losses. Tracing algorithms which can be considered direct to a good extent are implemented for these three methods. A case study based on a four bus system is provided and results obtained using MATLAB code is presented.

Keywords: *Deregulated Power System, Loss Allocation, Power Flow Tracing*

1.0 INTRODUCTION

Recent years have seen an almost unstoppable drive towards deregulation and privatization of the electricity supply industry worldwide. The old world of vertically integrated power utilities operating as regulated monopolies is collapsing almost everywhere to give way to a new world of competition and choice. All this is done in the hope that the market-oriented solutions will deliver increases in efficiency and decreases in prices. The first country to embark on the transformation path was Chile in 1982, followed by the United Kingdom (1990) and Norway (1991). In the following years a large number of countries from all parts of the world started to deregulate their electricity supply sectors. As a result, a number of regional electricity pools have been created and there is a new development in the restructuring process almost every month.

All the restructuring effort hinges on the idea of "unbundling", i.e. that electric energy can be separated commercially as a *product* from transmission as a *service*. In the past, electricity has been viewed as a product used only at the point of delivery and paid for in a single delivered tariff. But if it is possible to define and separate the trasport service, so that it can be provided separately from the electricity itself, electricity becomes a product that can be bought and sold and transported from place to place like any other product. Electricity markets are then opened to alternative producers and alternative purchasers.

Transmissin loss allocation is the process of assigning to each individual generation and load the responsibility of paying for a part of the system transmission losses. Although no power system variable is affected by this process, the revenue and payment reconciliation are dependant on the criterion adopted for this purpose. Transmission loss allocation is not an easy task. Even in a simple two-node system with one generator supplying a single load, loss allocation between the generator and the load has to be agreed upon as there is no physical measurement or mathematical method that determines the loss shared in a unique manner. In a real system, matters get more complicated because of two facts. The first is that the determintion of the line flows caused by each load through each transmission line has a good degree of arbitrariness. The second is that the transmission line loss is a nonlinear funtion of the line flow and hence cannot be separated between partial flows through the same line in a unique convincing way. Furthermore, if linearization techniques are used to allocate the flow of a given line to generators and demands, the cross terms associated with quadratic functions do not allow assigning directly losses to generators and consumers. These facts preclude the existence of a unique transmission loss allocation procedure based on different approaches, several methods have been proposed for transmission loss allocation.

Due to the fact that no unique or ideal procedure exists, any loss allocation algorithm should have most of the desirable properties stated below[2]:

1) To be consistent with the results of a power flow;
2) To depend on the amount of energy either produced or consumed;
3) To depend on the relative location in the transmission network;
4) To avoid volatility;
5) To be easy to understand;
6) To be simple to implement.

2.0 POWER FLOW TRACING

Tracing is a method of assigning flows in an electricity network to particular generator and load, assuming perfect mixing at each node. In this paper tracing is done through the proportional sharing principle, which has been used to develop different methods for loss allocation.

The methods that use this rule are briefly described below.

2.1 Graph Based Procedure

Tracing procedure for this method id based on the assumption of lossless lines in the network. The results of a converged power flow are used along with a linear proportional sharing rule to allocate the contribution of each generator for the loads and hence the trasmission losses between loads and generators.

2.2 Matrix Based Procedure

This method also is based on the assumption of lossless lines in the network. The results of converged power flow along with the proportional sharing principle are used to find the distribution matrix and the gross flows. The trasmission losses are determined between the loads and the generators.

2.3 Complex Power Flow Procedure

This involves the tracing of complex power flow with the help of results of converged power flow and proportional sharing rule. The extraction factors are determined for both real and reactive power and the unbundling line losses are found. The tracing is carried out including the transmission losses. Contribution of demand for the flows in each line and the losses are determined.

Using the proportional sharing principle, losses are allocated by linear procedures. It should be noted that a systematic application of this principle originates that all losses are allocated to demands. In order to allocate losses to generators, the method relies on a simple principle: losses associated with every line whose flow leaves a given bus are transferred to the lines whose flows enter the bus (or generation in that bus) proportionlly to the flows of those lines (whose flows enter the bus). It should be noted that a systematic application of this principle originates that all losses are allocated to generators.

A four bus system with five lines, two generators and two loads as shown in fig. 1 is used to compare the power flow tracing methods.

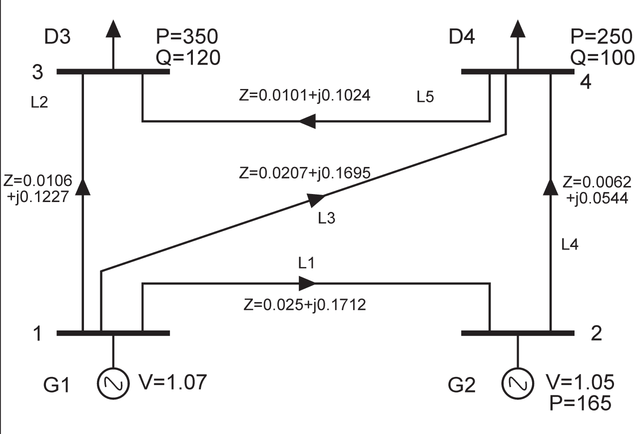

FIG. 1 FOUR BUS SYSTEM

The load flow is carried out using Newton-Raphson method and the individual flows along each line are computed. Sending end and receiving end line flows are tabulated for real power in Table 1.

TABLE 1

ACTIVE POWER FLOWS

Sending End Bus	Receiving End Bus	Sending End Flows	Receiving End Flows
1	2	62.759	-61.975
1	3	259.730	-250.497
1	4	132.584	-128.558
2	4	226.975	-222.514
4	3	101.072	-99.503

The real power loss and reactive power loss are calculated from load flow as 20.074 MW and 200.863 MVAR. Sending end and receiving end line flows are tabulated for reactive power in Table 2.

TABLE 2

REACTIVE POWER FLOWS

Sending End Bus	Receiving End Bus	Sending End Flows	Receiving End Flows
1	2	7.170	-1.203
1	3	179.62	-72.746
1	4	68.510	-35.537
2	4	166.766	-127.623
4	3	63.16	-47.254

3.0 GRAPH BASED ALGORITHM

The graph based procedure involves the application of proportional sharing rule at each node by traversing from source node to sink node or vice-versa. This requires establishing *domains*, or *dominions*, of each generator and processing the nodes according to their order in the directed graph made out of network flows. The network is assumed to be lossless. The algorithm is proposed for the acyclic directed graph of network flows i.e. contains no cycles. ***Downstream approch and upstream approach*** are the two approches for this method [4].

The upstream approach involves the traversing of nodes from sink to source node. In downstream approach, the traversing of nodes is from source to sink node. In the downstream approach, there are two versions of assuming the network to be lossless *gross flows, using average flows*. Similarly for upstream approach, the two versions used are *net flows and average flows*.

In the average flows version, the power flows are obtained by summing the average of magnitude of sending end and receiving end flows and half the line loss to the power injections at each terminal node of the line, where line loss is the difference between the magnitude of sending and receiving end flows. In the gross flows version, the power flows are assumed to be gross flows which can be defined as the sum of actual power flows and transmission losses accumulated in all lines supplying a given line or node. In the upstream approach using net flows, the line flows are assumed to be net flows which is defined as the difference between the actual flows and the transmission losses accumulated in all lines suppllying a given line or node.

In this paper, an algorithm for downstream approach using gross flows is implemented for the four bus symstem. In this approach, the tracing order is towards the flows, from the source node to the sink node downstream. At every node, gross nodal power, gross flow along the lines and the individual load contribution to each line is calculated.

3.1 Downstream Tracing Using Gross Flow

For downstream tracing, the concept of a gross power is implemented which would flow in the network as if it is fed with the actual generation and the network is lossless. A gross flow is equal to the sum of the actual flow and a transmission loss accumulated in all lines supplying a given line or node. As the gross flows are lossless, the value of the flow at the sending and the receiving end is the same. The downstream algorithm applied to the lossy real power flows shown would then give the following allocation:

Node 1: As this is a source node, power out flowing in lines L1, L2 and L4 comes exclusively from G1 and is equal to the power at the sending end of the line.

Node 2: The gross nodal power is equal to 62.759 + 165 = 227.759 supplied from G1 via L2 and from G2. The power out flowing in L3 has to be scaled up proportionally so that it is equal to the gross nodal power of 227.759. The composition of power out flowing in L3 can be calculated using the proportionality principle as (227.759/227.759)*62.759=62.759 from G1 and (227.759/227.759)*165=165 from G2.

Node 4: The gross nodal power is equal to (132.584 supplied from G1 via L4) + (227.759 supplied via L3) = 360.343. Adding the components originating from G1 gives the following composition of the gross nodal power: (132.584+62.759) = 195.343 supplied from G1 and 165 from G2. The gross nodal outflows have to be now scaled up proportionally as their sum must be equal to the gross nodal power. This gives the gross flow in L5 equal to 101.072 * [360.343 / (101.072+250)] = 103.7411 and the gross demand D4 equal to 250 * [360.343 / (101.072+250)] = 256.6019. The composition of the inflows in line L3 has been calculated above. The composition of the outflows is then:

D4: (256.6019/360.343)*195.343=139.1046 from G1 and (256.6019/360.343)*165=117.4973 from G2.

L5: (103.7411/360.343)*195.343=56.2384 from G1 and (103.7411/360.343)*165=47.5027 from G2.

Node 3: The gross nodal power is equal to the sum of gross nodal inflows, i.e. 259.730+103.7411 = 363.4711. Adding the inflowing components origination from the same gives the following decomposition: can be obtained by adding the shares supplied by lines L5 and L1 as: (56.2384+259.730) = 315.9684 from G1 and (47.5027+0) = 47.5027 from G2. As D3 is the only nodal outflow, these are also the components of D3. Note that gross demand D3 has also to be scaled up 350*(363.4711/350) = 363.4711 to be equal to the nodal inflows.

TABLE 3

ACTIVE POWER FLOWS AND LOSSES

	D3	D4	Total
G1	315.9684	139.1046	455.073
G2	47.5027	117.4973	165.000
Total	363.4711	256.6019	620.073
Actual	350.000	250.00	600.000
Loss	13.4711MW	6.6019MW	20.073

The contribution of each generator towards the load is calculated from the downstream tracing using gross flows and is shown in Table 3. The real power loss obtained is 20.073 MW out of which 13.4711 MW is allocated to D3 and 6.6019 MW is allocated to D4.

4.0 MATRIX BASED ALGORITHM

The matrix based procedure involves the application of proportional sharing rule in forming the distribution matrix and finding the lossless line flows contributed by the generators of the network. The network is assumed to be lossless. Thealgorithm is proposed for the acyclic directed flow of network flows i.e. contains no cycles. ***Downstream approach and upstream approach*** are the two approaches for matrix based algorithm [4].

The upstream approach involves the formation of downstream distribution matrix. In downstream approach, the formation of upstream distribution matrix is done. In the downstream approach, there are two versions of assuming the network to be lossless ***gross flows, using average flows***. Similarly for upstream approach, the two versions used are ***net flows, average flows***.

In the average flows version, the power flows are obtained by summing the average of magnitude of sending end and receiving end flows and half the line loss to the power injections at each terminal node of the line, where line loss is the difference between the magnitude of sending and receiving end flows.

In the gross flows version, the power flows are assumed to be gross flows which can be defined as the sum of actual power flows and transmission losses accumulated in all lines supplying a given line or node. In the upstream approach using net flows, the line flows are assumed to be net flows which is defined as the ddfference between the actural flows and the transmission losses accumulated in all lines supplying a given line or node.

An algorithm for **downstream approach using gross flows** is implemented for the four bus system and the results are obtained using MATLAB.

4.1 Downstream Tracing Using Gross Flows

Let us define an unknown *gross nodal power,* P_i^{gross} as a total power flow through node *i* which satisfies the Kirchoff's Current Law and which would flow if the network was fed with the actural generation and no power was lost in the network. Similarly, let $P_{i\text{-}j}^{gross}$ be an unknown *gross flow* in line *i-j* which would flow if no power was lost. Obviously,

$$\left| P_{i-j}^{(gross)} \right| = \left| P_{j-i}^{(gross)} \right| \qquad (1)$$

The grass nodal power, when looking at the inflows, can be expressed as

$$P_i^{(gross)} = \sum_{j \in \alpha_i^{(u)}} \left| P_{i-j}^{(gross)} \right| + P_{Gi} \text{ for } i = 1,2...n \qquad (2)$$

As $\left| P_{i-j}^{(gross)} \right| = \left| P_{j-i}^{(gross)} \right|$, the flow $P_{i-j}^{(gross)}$ can be replaced by $C_{ji}^{gross} P_j^{gross}$ where $C_{ji}^{(gross)} = \left| P_{j-i}^{(gross)} \right| / P_j^{(gross)}$. Normally the transmission losses are small so that it can be assumed that $\left| P_{j-i}^{(gross)} \right| / P_j^{(gross)} \cong \left| P_{j-i}^{(gross)} \right| / P_j$, where P_{j-i} is the actual flow through node j in line j-i and P_j is the actual flow through node j. This corresponds to assuming that distribution of gross flows at any node is the same as distribution of actual flows. This is the only approximating assumption of this method. Using this assumption eqn (2) can be re-written as

$$P_i^{(gross)} - \sum_{j \in \alpha_i^{(u)}} \left(\left| P_{j-i} \right| / P_j \right) P_j^{(gross)} = P_{Gi} \qquad \text{(or)}$$

$$[A_u]\,[P_{gross}] = [P_G] \quad (3)$$

Where,

A_u is the $(n \times n)$ upstream distribution matrix.

P_{gross} is the vector of nodal through-flows

P_G is the vector of nodal generations.

The matrix is called upstream (despite tracking the flow downstream) as $\alpha_i^{(u)}$ corresponds to all nodes upstream from node i. The (i,j) element of A_u is equal

$$[A_u]_{ij} = \begin{cases} 1 & \text{for } i=j \\ -C_{ji} = -\left|P_{j-i}\right|/P_j & \text{for } j \in \alpha_i^{(u)} \\ & \text{otherwise} \end{cases}$$

Note that A_u is sparse and non-symmetric matrix. If A_u^{-1} exists then

$$[P_{gross}] = [A_u^{-1}]\,[P_G]$$

Pgross is the unknown vector of gross nodal flows. As A_u and P_G are known, the solution of eqn (3) will give the unknown gross nodal flows. Once the gross nodal flows have been determined, the gross line flows and gross demands can also be found using the proportional sharing principle. The gross outflow from node i in line i-l is

$$\left|P_{i-l}^{gross}\right| = \left(P_i^{gross}\right) * \left(\left|P_{i-l}^{gross}\right| / P_i^{gross}\right) \cong \left|P_{i-l}\right| \sum_{k=1}^{n} [A_u^{-1}]_{ik}\, P_{Gk}$$

For all $1l \in \alpha_i^{(d)}$ (4)

While the gross demand at node i can be calculated as

$$\left|P_{Di}^{gross}\right| = \left(P_i^{gross}\right) * \left(\left|P_{Di}^{gross}\right| / P_i^{gross}\right) \cong \left(P_{Di}/P_i\right) * P_i^{gross} = \left(P_{Di}/P_i\right) \sum_{k=1}^{n} [A_u^{-1}]_{ik}\, P_{Gk} \quad (5)$$

This equation is especially important as it shows what would be the load demand at a given node if a lossless network was fed with the actual generation. Hence the difference between the gross demands and the actual demand.

$$\Delta P_{Di} = P_{Di}^{gross} - P_{Di} \quad (6)$$

Eqn (6) gives the loss which is attracted by power flowing from all the generators to a particular load. Let us apply this algorithm to the real power flow of four bus system.

Eqn (3) gives:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -62.759/455.07 & 1 & 0 & 0 \\ -259.730/455.07 & 0 & 1 & -101.07/351.07 \\ -132.584/455.07 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} 455.073 \\ 165.0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this equation gives the following values of gross nodal powers:

$$[P_{gross}] = \begin{bmatrix} 455.074 \\ 227.759 \\ 363.4711 \\ 360.344 \end{bmatrix}$$

The gross load demands are

$D_3^{(gross)}$ = (350/350)*363.4711 = 363.4711 and

$D_4^{(gross)}$ = 360.344*(250/351.072) = 256.6019

Hence the loss apportioned to D_3 *is* equal to 13.4711 while the loss apportioned to D_4 is 6.6019. The obtained results are shown in Table 4.

TABLE 4

ACTIVE POWER FLOWS AND LOSSES

	D3	D4	Total
G1	315.9684	139.1046	455.073
G2	47.5027	117.4973	165.000
Total	363.4711	256.6019	620.073
Actual	350.000	250.00	600.000
Loss	13.4711MW	6.6019MW	20.073

The sum of the elements in each of the generator rows gives the actual generation. The sum of the elements in each of the load columns gives the gross demand for each of the loads. The difference between the gross and actual demand gives the transmission loss associated with supplying a paricular load.

5.0 COMPLEX POWER FLOW TRACING ALGORITHM

This method is based on tracing the complex power flow through the network and determing the share of each load on the flow and losses through each line [3]. Transmission losses are taken into consideration during power flow tracing. Unbundling line losses is carried out using an equation, which has a physical basis, and considers the coupling between active and reactive power flows as well as complex losses are considered simultaneously and not separately.

In this algorithm, the transmission line is represented by two sets of extraction factors, one at the sending end and the second at the receiving end of the line. Each set contains two extraction factors, one for active power wheras the second is for reactive power. The sending end factors are negative. Different signs are used for extraction factors at the two ends of the line so that these extraction factors will have information about the power flow directions.

FIG. 2 TRANSMISSION LINE MODEL

Consider the line, l, shown in fig. 2. Since the extraction factors for active power and reactive power are defined in the same way, f is used to denote both active and reactive power flows through the line. That is to say f_{il} is the flow, either active or reactive, through line l out of node i. For the purpose of clarity, it is required to define the net injection at a node, M, and the total flow F passing a node.

Let node i be the sending end and node j be the receiving end of the line, and the extraction factors are defined as follows:

At the sending end, the extraction factor is defined as the fraction of the net power passing that node, i, which is extracted by line. According to the proportional sharing principle, this factor represents the contribution of the flow through line into the flows through all lines preceding it in the upstream direction. At the receiving end, the extraction factor iis defined as the fraction of the load at that node, j, which is fed through line l. If node j is not a load node, then no power will be extracted at this node from any line. An algorithm is implemented for the four bus system and the results are obtained using MATLAB.

6.0 ALGORITHM

The tracing algorithm [3] starts at the receiving end of an end line, moving along the feed paths in the upstream direction to generators determining the partial flow caused by the load through each line on the feeding path.

Line factor matrices

From the line flow data, form the line flow matrices FP and FQ.

$$FP=[P_{jl}] \text{ and } FQ=[Q_{jl}] \tag{7}$$

The Mi matrix for FP and FQ matrices using

$$M_i = \sum_{l \in \psi_i} f_{il} \tag{8}$$

i – Sending end node

M_i – Net injection at a node i

f_{il} – Power flow either active or reactive through the line 1 out of i

Ψ_i – set of all lines incident to node I

Now, calculate the F_i matrix for FP and FQ matrices using

$$F_i = \begin{cases} \sum_{l \in \psi_{Pi}} f_{il} & M_i \geq 0 \\ \sum_{l \in \psi_{ni}} -f_{il} & M_i < 0 \end{cases} \tag{9}$$

Ψ_{pi} - set of lines carrying outflows from node i

Ψ_{ni} - set of lines carrying inflows to node i

Extraction factor matrices

For sending end i, it is the fraction of net power passing through that node i, which is extracted by line 1.

$$K_{fil} = f_{il}/F_i \tag{10}$$

For receiving end j, it is the fraction of load at that node j, which is fed through line 1.

$$F_{fil} = \begin{Bmatrix} M_j / F_j & M_j < 0 \\ -\alpha & M_j \geq 0 \end{Bmatrix} \tag{11}$$

Where $\alpha = 10^{-10}$; j = receiving end.

Using these elements form the extraction factor matrices KP and KQ.

For each line, determine the receiving end to sending end transfer matrix T using

$$[T_l] = \begin{bmatrix} T_{l,pp} & T_{l,pq} \\ T_{l,qp} & T_{l,qq} \end{bmatrix} \tag{12}$$

$T_{l,pp} = (P_{jl} / P_{il}) + (1-(P_{jl} / P_{il}))\ ((P_{jl} / |\ S_{jl}\ |)^2)$

$T_{l,pq} = (1-(P_{jl} / P_{il}))\ ((Q_{jl} / |\ S_{jl}\ |)^2)$

$T_{l,qp} = (1-(Q_{jl} / Q_{il}))\ ((P_{jl} / |\ S_{jl}\ |)^2)$

$T_{l,qq} = (Q_{jl} / Q_{il}) + (1-(Q_{jl} / Q_{il}))\ ((Q_{jl} / |\ S_{jl}\ |)^2)$

As long as there are still positive elements in KP and KQ, do the following:

END LINE: A line with its negative extraction factors sum up to -1.

- Pick an end line, let this be column 1.
- Find the sending node of 1, row, i and set the positive to zero.
- For each negative element in i, that represents a line, m, preceding line in the upstream direction.

For each negative at line 1, j, determine the extraction factors from the line m, for both active power and reactive power.

$Kp_{jm} = Kp_{jm} + Kp_{il}\ (\ T_{l,pp}Kp_{jl} + T_{l,pq}Kq_{jl})$

$Kq_{jm} = Kq_{jm} + Kq_{il}\ (\ T_{l,qp}Kp_{jl} + T_{l,qq}Kq_{il})$

The procedure ends when all positive elements in KP and KQ are set to zero. The sum of each column of KP and KQ will be -1. Applying this algorithm to the four bus system, we get. The flow matrices for this system using eqn (7), (8), (9) will be as follows.

FP =

62.76	259.73	132.58	0.0	0.0
-61.98	0.0	0.0	226.98	0.0
0.0	-250.5	0.0	0.0	-99.50
0.0	0.0	-128.55	222.51	101.07

M_i	F_i
455.07	455.07
165.00	226.97
-350.0	350.00
-250.0	351.07

FQ =

7.17	179.62	68.51	0.0	0.0
-1.20	0.0	0.0	166.76	0.0
0.0	-72.74	0.0	0.0	-47.25
0.0	0.0	-35.54	-127.62	63.16

M_i	F_i
255.30	255.30
165.56	166.76
-120	120
-100	163.16

Using the extraction factor eqn (10) and (11) for sending and receiving ends, KP and KQ are formed as follows:

$$KP = \begin{bmatrix} 0.14 & 0.57 & 0.29 & 0 & 0 \\ -\alpha & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & -0.71 & -0.71 & 0.29 \end{bmatrix}$$

$$KQ=\begin{bmatrix}0.03 & 0.70 & 0.27 & 0 & 0\\ -\alpha & 0 & 0 & 1 & 0\\ 0 & -1 & 0 & 0 & -1\\ 0 & 0 & -0.61 & -0.61 & 0.39\end{bmatrix}$$

The transfer matrices of the lines are calculated using eqn (12)

T1 =

1.0	0
0.8319	0.1681

T2 =

0.9972	0.0028
0.5487	0.4513

T3 =

0.9978	0.0022
0.4471	0.5529

T4 =

0.9951	0.0049
0.1766	0.8234

T5 =

0.9971	0.0029
0.2055	0.7945

The proposed tracing algorithm starts with end lines that have negative extraction factors that sum up to -1. Checking KP and KQ shows that there are two end lines: the second and the fifth lines. Starting with the second column, its sending is node 1. There are no negative elements in this row, which means that this line draws no power from any other lines, but it takes its power directly from a source node. Therefore, the positive element Kp_{12} and Kq_{12} are set to zeros without affecting the other lines.

Now for line 5, 1=5, its sending end is node 4, i=4, which has two negative elements KP_{43} and KP_{44}, m=3,4. The negative element in column 5 is Kp_{35}, j=3.

Applying for KP,

$$Kp_{33} = Kp_{33} + Kp_{45}\ (T_{5,pp} * Kp_{35} + T_{5,pq} * Kq_{35})$$
$$= 0 + 0.29\ (0.9971 * -1 + 0.0029 *-1) = \mathbf{-0.29}$$

Applying for KQ,

$$Kq_{33} = 0 + Kq_{45}\ (T_{5,qp} * Kp_{35} + T_{5,qq} * Kq_{35})$$
$$= 0 + 0.39\ (0.2055 * -1 + 0.7945 *-1) = \mathbf{-0.39}$$

Kp_{34} and Kq_{34} are calculated in a similar way. After processing line 2 and line 5, KP and KQ become

$$KP=\begin{bmatrix}0.14 & 0 & 0.29 & 0 & 0\\ -\alpha & 0 & 0 & 1 & 0\\ 0 & -1 & -0.29 & -0.29 & -1\\ 0 & 0 & -0.71 & -0.71 & 0\end{bmatrix}$$

$$KQ=\begin{bmatrix}0.03 & 0 & 0.27 & 0 & 0\\ -\alpha & 0 & 0 & 1 & 0\\ 0 & -1 & -0.39 & -0.39 & -1\\ 0 & 0 & -0.61 & -0.61 & 0\end{bmatrix}$$

Now lines 3 and 4 become end lines. The sending end of line 3 is a source node, node 1. Therefore, it will not affect other lines; Line 4 is treated in the same way as line 5 above. The affected elements are Kp_{31}, Kp_{41}, Kq_{31} and Kq_{41}.

$$Kp_{31} = 0 + 1\ (0.9951 * -0.29 + 0.0049 * -0.39)$$
$$= \mathbf{-0.29}$$

$$Kq_{31} = 0 + 1\ (0.1766 * -0.29 + 0.8234 * -0.39)$$
$$= \mathbf{-0.37}$$

$$Kp_{41} = 0 + 1\ (0.9951 * -0.71 + 0.0049 * -0.61)$$
$$= \mathbf{-0.71}$$

$$Kq_{41} = 0 + 1\ (0.1766 * -0.71 + 0.8234 * -0.61)$$
$$= \mathbf{-0.63}$$

After processing lines 3 and 4, KP and KQ become,

$$KP=\begin{bmatrix}0.14 & 0 & 0 & 0 & 0\\ -\alpha & 0 & 0 & 0 & 0\\ -0.29 & -1 & -0.29 & -0.29 & -1\\ -0.71 & 0 & -0.71 & -0.71 & 0\end{bmatrix}$$

$$KQ=\begin{bmatrix} 0.03 & 0 & 0 & 0 & 0 \\ -\alpha & 0 & 0 & 0 & 0 \\ -0.37 & -1 & -0.39 & -0.39 & -1 \\ -0.63 & 0 & -0.61 & -0.61 & 0 \end{bmatrix}$$

Line 1 is now an end line with negative extraction factors that sum up to -1. Its sending end is node 1 and it will not affect other lines. The positive element of column 1 as well as -α can now be removed and the final matrices KP and KQ will be

$$KP=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -0.29 & -1 & -0.29 & -0.29 & -1 \\ -0.71 & 0 & -0.71 & -0.71 & 0 \end{bmatrix}$$

$$KQ=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -0.37 & -1 & -0.39 & -0.39 & -1 \\ -0.63 & 0 & -0.61 & -0.61 & 0 \end{bmatrix}$$

Elements of KP and KQ define the shares of each load node in the power flow at the receiving ends of the system branches. Knowing the flow caused by a load node at both ends of a branch, losses caused by this node through this branch can be calculated.

TABLE 5

ACTIVE POWER FLOWS AND LOSSES

Line Bus	Active Power Flow at Sending End (MW)				
	1-2	**1-3**	**1-4**	**2-4**	**4-3**
3	18.22	260.70	38.35	65.72	101.61
4	44.97	0	94.85	162.56	0

Line Bus	Active Power Flow at Receiving End (MW)				
	1-2	**1-3**	**1-4**	**2-4**	**4-3**
3	17.87	251.34	37.13	64.64	100.00
4	44.10	0	91.84	159.89	0

Line Bus	Active Power Loss (MW)					
	1-2	**1-3**	**1-4**	**2-4**	**4-3**	**Total**
3	0.35	9.36	1.22	1.08	1.61	13.62
4	0.87	0	3.01	2.67	0	6.55

7.0 COMPARISON OF RESULTS

The following Table provides the real power losses allocated using the three compared procedures for the four bus test system.

TABLE 6

COMPARED RESULTS

Load Bus	Graph Method	Matrix Method	Complex Tracing Method
3	13.4711	13.446	13.62
4	6.6019	6.584	6.55

8.0 CONCLUSION

From the comparison of case study, the following conclusions are drawn.

1) Graph based method is simple for the computation of losses if the directed graph is acyclic.
2) Although graph method is simpler, matrix based method is computationally equivalent to it and does not require time consuming matrix inversion as it is performed recursively.
3) Complex power flow tracing method determines the share of each load through each individual line.

After analyzing the methods, final recommendations are as follows.

1) Graph based method is not advisable for large network and fails for those with circular flows.
2) Matrix based method is acceptable for the network with cyclic flows as it gives meaningful solution.
3) Complex power flow tracing method is recommendable as it determines active and reactive flows and losses simultaneously. No matrix inversion and additional nodes for loss representation are required.

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