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A Simple and General method to Perform Large-Scale Series/Parallel Reduction of Proximally Coupled Two-terminal Networks by Employment of the Primitive Impedance/Admittance Matrices

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The process of reduction of series/parallel combinations involving an arbitrarily large number of magnetically coupled two-terminal networks containing a diverse combination of circuit elements is a conceptually fundamental task and yet has received only scant attention in relevant literature, the only attempts in this direction being limited to a few ad hoc and laboured developments featuring extremely elementary topologies while furnishing meagre, if any, scope for mathematical extension. This paper proposes a powerful tool wherewith large-scale series/parallel reductions of networks with proximal coupling may be achieved in an efficient and facile manner, without any specific limit on the number of constituents, thereby ensuring the generality of the approach. The primitive impedance/admittance matrices are employed for this purpose in their time-domain operational form, which ensures the retention of the most general form of the interrelationships between the terminal voltages and currents of the individual blocks as well as their combination. The presented method has been demonstrated with numerous examples involving inductor networks and inductor-resistor combinations in series, parallel,and some series/parallel topologies.

Keywords: *Network reduction, proximal coupling, primitive impedance/admittance matrix, series/parallel reduction*

1.0 INTRODUCTION

The reduction of an interconnected passive network at a port of access (say, at a terminal-pair) to an ‘equivalent’ impedance or admittance (in the most general sense of these terms) is a conceptually fundamental task in circuit theory, and has received more than a century of detailed and meticulous attention from earnest inquirers in this field. The outcome of such inquiries is copious and rich in details, insofar as the reduction of topological (interconnection-originated) relationships resident in any given network is concerned.

Now then, circuit theory also provides for a second type of relationship that could exist within and between topological blocks, to wit – that due to sharing a common physical neighbourhood. One instance of such a relationship has its origins in magnetic proximity.In circuit theory, this behavioural aspect is attributed exclusively to inductors and is quantified as a measure of the extent of such a proximity-originated ‘coupling’ between inductors, via the agency of ‘mutual’ inductance. This appellation stems from the fact that such a relationship that is based on magnetic proximity must evidently be a mutual one. This proximal relationship exists regardless of the manner of electrical interconnection; indeed, it

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manifests itself completely independent of any topological binding whatsoever.Any general and meaningful approach towards network reduction must therefore contend with this duo of relationships – topological and proximal – in a conjoint and comprehensive fashion.

Whereas the treatment of topological reduction has received a fulsome treatment by several authors over several decades, it is indeed baffling to the assiduous inquirer who is met with a near total paucity of adequate literature concerning the problem of network reduction involving proximal coupling. This conceptually fundamental topic appears to be strangely and summarily neglected so as to leave a pedagogical lacuna of glaring dimensions. A systematic survey of classic and contemporary authorities [1-41] bears ample testimony to the truth of this observation. Analytical treatment has been, by and large,ubiquitously limited to the trivial case of two coupled inductors in series arrangement. Some attempts at network reduction beyond the above ubiquitous case may be summarized as follows:

1. The case of two coupled inductors in parallel appears in references [1-5]. However, it is relegated in a decidedly indifferent manner to chapter-end exercises in [3-5]. References [1-2] offer an ad-hoc and cumbersome approach which is not amenable for extension to a higher number of coupled elements.
2. Reference [1] deals with the case of two coupled R-L branches in parallel in a rigid and somewhat straitlaced treatment, which is incapable of facile extension and is limited in scope to the sinusoidal steady state.
3. Reference [2] deals with the case of three coupled inductors in series by an approach of an 'effective' inductance, which could be misleading and is exceedingly inconvenient from the point of view of generalization.

The rest of the surveyed works [6-41] devote no more attention to this problem than a cursory mention the trivial case. Given the fundamental nature of the problem, it is disheartening and perplexing to countenance this near absence of adequate treatment concerning thereto.

This paper attempts to supply this need by advancing a simple and general approach towards network reduction, which is capable of treating the twin relationships – topological and proximal – in an efficiently overlaid fashion by the employment of the primitive impedance (or admittance) matrices and adroit, yet simple,mathematical manipulations thereupon. The developed method is eminently scalable to any size, and with suitable modifications, to any general network topology.

2.0 ANALYTICAL DEVELOPMENT

The two-terminal networks are characterized by their terminal variables – namely, voltages and currents expressed as functions of time. It will be convenient to retain this time-domain representation of the terminal variables and to interrelate these via suitable rational functions. This approach obviates the need for domain transformations and also prevents the modelling from being restricted to a very specific set of operating conditions. To achieve this generalization, use is made of the Heaviside differential operator given by

$$P \equiv \frac{d}{dt} \qquad ...(1)$$

Employment of this operator ensures the retention of the parent domain (viz. time t) of the variables of interest, while simultaneously facilitating the algebraic manipulation of the underlying differential equations.

The terminal variables – such as voltage $V_k\,(t)$ and current $i_k\,(t)$ hall be denoted plainly as V_k and i_k hereinafter unless otherwise specified.

2.1 The Primitive Network

A set of n_L proximally coupled two-terminal networks is shown in Figure 1.

These networks are topologically unconnected but possess coupling due to magnetic proximity. This state of existence of the n_L networks is referred to as 'primitive' [42-43]. The performance equation of this set of networks may be written as mathematically coupled differential-algebraic equations using the terminal voltages and currents. This description is called as the primitive description of the set of networks and may be expressed in either the operational impedance form

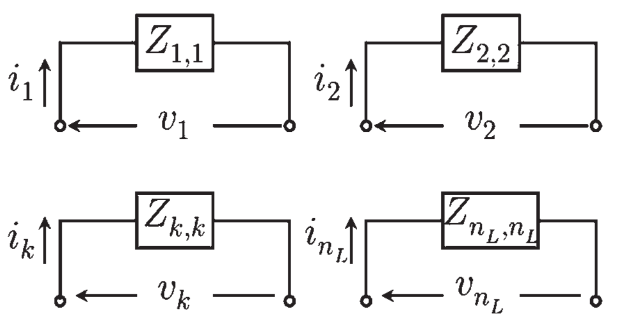

FIG. 1 PRIMITIVE DISPOSITION OF n_L PROXIMALLY COUPLED TWO-TERMINAL NETWORKS

$$\underline{v} = Z_P \underline{i} \qquad ...(2)$$

or in the operational admittance form

$$\underline{i} = Y_P \underline{v} \qquad ...(3)$$

where $\underline{v} = [v_1 v_2 \cdots v_{n_L}]^T$ and $\underline{i} = [i_1 i_2 \cdots i_{n_L}]^T$ are $n_L x1$ column vectors and

$$Z_p = \begin{bmatrix} Z_{1,1} & \cdots & Z_{1,n_L} \\ \vdots & \ddots & \vdots \\ Z_{n_L,1} & \cdots & Z_{n_L,n_L} \end{bmatrix}$$

is the $n_L x n_L$ primitive impedance matrix of the given set of n_L coupled two-terminal networks. The diagonal elements $Z_{k,k}$ of this matrix are the 'self' impedances of the networks whereas the off-diagonal elements $Zk,j(k \neq j)$, are the impedances 'mutual' to networks k and j, which characterize the extent of coupling between elements k and j due to magnetic proximity. It may be noted that for all k and j,

$$Z_{k,j} = Z_{j,k}\,; k, j = 1,2, \cdots n_L$$

That is, Zp is a symmetric matrix.

All the elements of the matrix Zp areopen-circuit impedances; this matrix reduces to a diagonal form, if none of the Zp networks were to be magnetically coupled. All the elements of Zp are,in general, rational functions of the operator p.

The matrix $Yp=[Zp]^{-1}$ is the primitive admittance matrix, obtained either by inversion of Zp or by direct measurement as the short-circuit admittance matrix.This is also a symmetric matrix. The elements of Yp are likewise rational polynomials.

2.2 Conditions Imposed by Topology

When the constituent units comprising the primitive network are subject to interconnection in a specific manner, an additional set of relationships would be brought to bear upon the terminal variables $\underline{v}$ and $\underline{i}$ These topological relationships impose an overlay of constraints over and above the 'primitive' ones, that is, over those due to proximal coupling. In any given 'interconnected' situation, therefore, it is necessary to consider the two sets of relationships in conjoint fashion. As a preparatory step in this process, the primitive matrices Zp and Yp might be required to undergo a modification or adjustment to correctly account for the (possibly) altered disposition of the senses of the terminal variables from those of the primitive network. This leads to the formulation of the altered primitive matrices Z and Y, these being obtained from the 'reported' primitive matrices Zp and Yp merely via necessary changes effected in the sign of the off-diagonal elements. With these changes, the modified primitive relationships may be expressed, for a given topology, in impedance form, as

$$\underline{v} = Z\underline{i} \qquad ...(4)$$

and, in admittance form, as

$$\underline{i} = Y\underline{v} \qquad ....(5)$$

In the development that follows, matrices Z and Y are taken to mean the suitably adjusted versions of the matrices Z_p and Y_p. This process is illustrated later on with number-imposed examples.

This manner of overlaying (or superimposing) the topological constraints upon the primitive relationships governing the coupled networks isexplained belowwith reference to someelementary – butby no means trivial – interconnections.

2.3 Series Connection of n_L Coupled Networks and Determination of the Equivalent Impedance Thereof

The n_L coupled networks are shown connected in series in Figure 2. The terminal voltage and current for the series combination are v_t and i_t. The equivalent impedance $Z_{eq} = \frac{v_t}{i_t}$ of the series combination is sought.

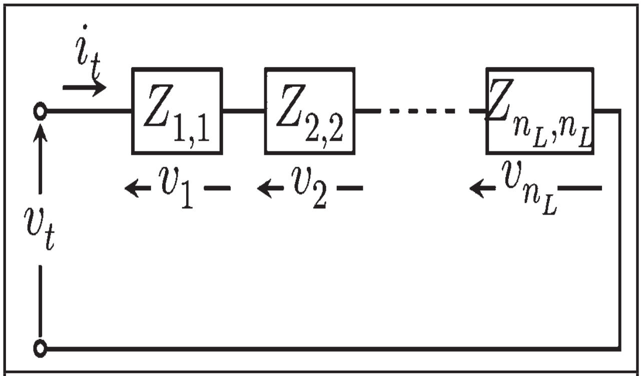

FIG. 2 SERIES CONNECTION OF n_L PROXIMALLY COUPLED TWO-TERMINAL NETWORKS

The topological conditions imposed upon the terminal variables $\underline{v}$ and $\underline{i}$ are

$$i_k = i_t, \qquad k = 1,2,\cdots,n_L\ ;\ v_t = \sum_{k=1}^{n_L} v_k$$

Introducing an $n_L x1$ column vector whose elements are all unity, that is,

$$\underline{1} = [1\ 1\ 1 \cdots 1]^T,$$

The series connection conditions may be more concisely expressed as

$$\underline{i} = \underline{1} i_t\ ;\ \ \underline{1}^T \underline{v} = v_t \qquad \text{...(6)}$$

Premultiplying (4) by $\underline{1}^T \underline{v}$ and applying (6) thereon, one obtains

$$v_t = \underline{1}^T Z \underline{1} i_t \qquad \text{...(7)}$$

Equation (7) is of the form $v_t = Z_{eq} i_t$ where Z_{eq} is the equivalent impedance of this series combination, leading to

$$Z_{eq} = \underline{1}^T Z \underline{1} \qquad \text{...(8)}$$

It is evident that the triple vector-matrix product $\underline{1}^T Z \underline{1}$ is a scalar quantity and is simply the sum of all the elements of the adjusted primitive matrix Z, thus simplifying the process of network reduction to a simple summation of the matrix elements. It may also be noted that this expression for equivalent impedance is equally valid under conditions of absence of coupling as well: in such a case, the summation is that of only the diagonal elements, the off-diagonal elements of Z then being null.

A useful corollary of the above result is the expression for the voltage v_k across the k^{th} two-terminal network:

$$\frac{v_k}{v_t} = \frac{\underline{1}^T \underline{Z}_k}{\underline{1}^T Z \underline{1}} = \frac{\underline{Z}_k^T \underline{1}}{\underline{1}^T Z \underline{1}} \qquad \text{...(9)}$$

where $\underline{Z}_k$ is the k^{th} column which is also the transpose of the k^{th} row) of Z. The equations (8) and (9) summarize the results of this series connection. It may also be observed that the simple case of the uncoupled condition is now expressible as a special (and the most elementary) case of the above results.

2.4 Parallel Connection of n_L oupled Networks and Determination of the Equivalent Impedance There of

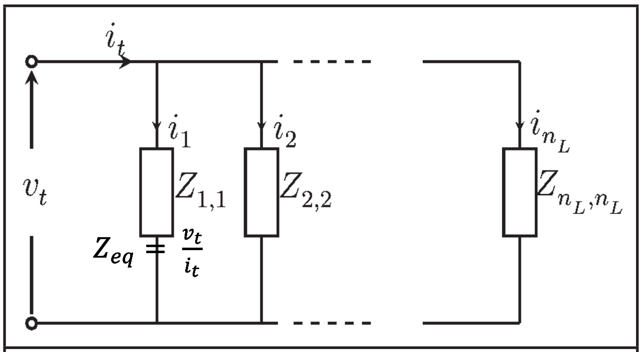

FIG. 3 PARALLEL CONNECTION OF n_L PROXIMALLY COUPLED TWO-TERMINAL NETWORKS

The n_L coupled networks are shown connected in parallel in Figure 3. The terminal voltage and current for the parallel combination are v_t and i_t. The equivalent impedance $Z_{eq} = \frac{v_t}{i_t}$ of the parallel combination is sought.

The topological conditions imposed upon the terminal variables $\underline{v}$ and $\underline{i}$ by this parallel connection are

$$v_k = v_t, \qquad k = 1,2,\cdots,n_L\ ;\ i_t = \sum_{k=1}^{n_L} i_k$$

which, in a more concise fashion, may be expressed as

$$v = 1v_t\ ;\quad 1^T i = i_t \qquad \text{...(10)}$$

The admittance form is better suited in this case for simplification. Premultiplying (5) by $\underline{1}^T$ and applying (10) thereon, one obtains

$$i_t = \underline{1}^T \mathrm{Y} \underline{1} v_t \qquad \text{...(11)}$$

Equation (11) is of the form $i_t = Y_{eq} v_t$, where Y_{eq} is the equivalent admittance of this parallel combination, and thus

$$Y_{eq} = \underline{1}^T \mathrm{Y} \underline{1} \qquad \text{...(12)}$$

Here too, it is manifest that the triple vector-matrix product $\underline{1}^T \mathrm{Y} \underline{1}$ is a scalar quantity and is simply the sum of all the elements of the adjusted primitive matrix Y, thus simplifying the process of network reduction to a simple summation of the matrix elements. The equivalent impedance Z_{eq} of ation may then be written as

$$Z_{eq} = \frac{1}{Y_{eq}} = \frac{1}{\underline{1}^T \mathrm{Y} \underline{1}} \qquad \text{...(13)}$$

A useful corollary of the result contained in (12) is the expression for the current i_k through the k_{th} two-terminal network:

$$\frac{i_k}{i_t} = \frac{\underline{1}^T \underline{\mathrm{Y}_k}}{1^T \mathrm{Y} 1} = \frac{\underline{\mathrm{Y}_k^T}\, \underline{1}}{1^T \mathrm{Y} 1} \qquad \text{...(14)}$$

where $\boldsymbol{n_L}$ is the k_{th} column (which is also the transpose of the k_{th} row) of Y. The equations (12), (13), and (14) summarize the results of thisparallel connection.It may also be observed that the simple case of the uncoupled condition is now expressible as a special (and the most elementary) case of the above results.

2.5 Specific Particularization of the General Results to a Group of n_L Coupled Inductors and the Definition of Levitance

A set of n_L proximally coupled inductors is shown in Figure 4.

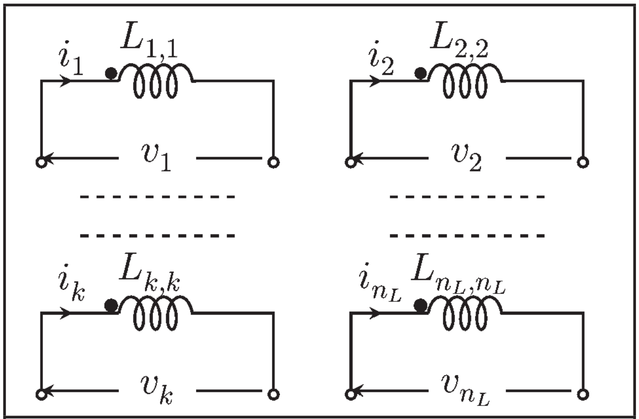

FIG. 4 PRIMITIVE DISPOSITION OF n_L PROXIMALLY COUPLED INDUCTORS

The state of affairs is similar to that shown in Figure 1 and detailed in §2.1.

Formulating, therefore, on similar lines, the terminal variables of these inductors are related as

$$v = \mathrm{L_P}\, pi \qquad \text{...(15)}$$

or, in the inverted form, as

$$p\underline{i} = \Gamma_\mathrm{P} \underline{v} \qquad \text{...(16)}$$

In (15) and (16), $\underline{v}$ and $\underline{i}$ are the terminal voltage and current vectors, and

$$\mathrm{L_p} = \begin{bmatrix} L_{1,1} & \cdots & L_{1,n_L} \\ \vdots & \ddots & \vdots \\ L_{n_L,1} & \cdots & L_{n_L,n_L} \end{bmatrix}$$

is the $n_L x n_L$ inductance matrix which is real, symmetric, and positive definite. The subscript P denotes the primitive nature of the characterization, Further,

$$\Gamma_\mathrm{P} = [\mathrm{L_P}]^{-1}$$

is the inverse of the inductance matrix. Each element of this T_p matrix has the dimensions of reciprocal inductance. It may be noted that a ‘single-dot’ convention is used in Figure 4, which conveniently harmonizes with the terminal-current senses and the specified entries in the primitive inductance matrix. This is in concordance with the spirit of the highly edifying observation ventured in this regard by Guillemin [9].

In order to avoid a possible conflation with the scalar ‘reciprocal of inductance’, and also since no specific name has been accorded in literature to this particular quantity, the authors deem fit to coin a term to negotiate this piquant situation. The term ‘Levitance’ (etymologically from ‘levity’ as contrasted with ‘gravity’) has been used here by the authors to denominate a quantity such as $t_{k,j}$ which is any element of t_p. It may be noted that, for a single isolated inductor of inductance value L_k, the levitance T_k becomes merely the reciprocal of L_k. Thus T_p may be christened as the‘levitance matrix’ of the group of n_L coupled inductors shown in Figure 4.

Considerations of the constraints brought about by any given interconnection will require the matrix L_p (and consequently T_p) to be updated via suitable modifications effected in the sign of its off-diagonal elements in much the same manner as detailed for Z_p in §2.2. This results in the formulation of the altered inductance matrix, which now reflects the necessary changes effected in the sign of its off-diagonal elements to incorporate the (possibly) changed senses of the terminal currents for the given topological configuration. These updated relationships may be expressed as

$$\underline{v} = \mathrm{L}\, p\underline{i} \qquad \text{...(17)}$$

and in the inverted form as

$$p\underline{i} = \Gamma \underline{v} \qquad \text{...(18)}$$

2.6 Series Connection of n_L Coupled Inductors and Determination of the Equivalent Inductance Thereof

The n_L coupled inductors are shown connected in series in Figure 5.

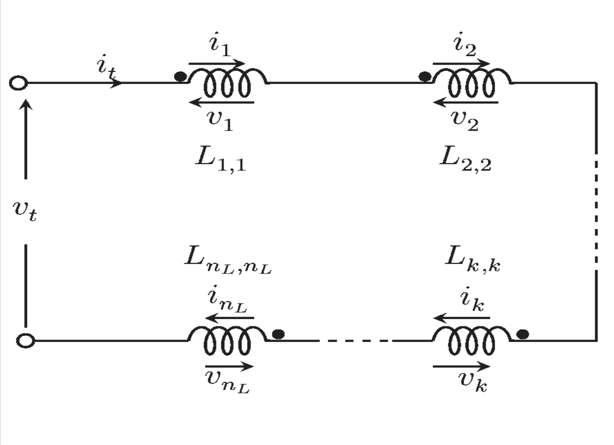

FIG. 5 SERIES CONNECTION OF n_L PROXIMALLY COUPLED INDUCTORS

The terminal voltage and current for the series combination are v_t and i_t. The equivalent inductance L_{eq} of this series combination is sought.

The topological conditions imposed by the series connection are those given by (6). Premultiplying (17) by $\underline{1}^T$ and applying (6) thereon, one obtains

$$v_t = \underline{1}^T \mathrm{L} \underline{1} p i_t \qquad \text{...(19)}$$

Equation (19) is of the form $v_t = L_{eq} p i_t$, where L_{eq} is the equivalent inductance of the series combination, and thus

$$L_{eq} = \underline{1}^T \mathrm{L} \underline{1} \qquad \text{...(20)}$$

is the equivalent inductance of the series combination of the n_L coupled inductors, which is obtained by a direct summation of all the elements of the adjusted inductance matrix L.

Further, the voltage across the k^{th} inductor may be given as

$$\frac{v_k}{v_t} = \frac{\underline{1}^T \mathrm{L}_k}{\underline{1}^T \mathrm{L} \underline{1}} = \frac{\mathrm{L}_k^T \underline{1}}{\underline{1}^T \mathrm{L} \underline{1}} \qquad \text{...(21)}$$

where $\underline{1}^T$ is the k^{th} columnwhich is also the transpose of the k^{th} row) of L. The equations (20) and (21) summarize the results of this series connection. It may also be observed that the simple case of the uncoupled condition is now expressible as a special (and the most elementary) case of the above results.

2.7 Parallel Connection of n_L Coupled Inductors and Determination of the Equivalent Inductance Thereof

The n_L coupled inductors are shown connected in parallel in Figure 6. The terminal voltage and current for the series combination are v_t and i_t The equivalent inductance L_{eq} of this parallel combination is sought.

The topological conditions imposed by the parallel connection are those given by (10). Premultiplying (18) by $\underline{1}^T$ and applying (10) thereon, one obtains

$$pi_t = \underline{1}^T\Gamma\underline{1}v_t \qquad ...(22)$$

Equation (22) is of the form $pi_t = \Gamma_{eq}i_t$,, where Γ_{eq} is the equivalent levitance of the series combination, and thus

$$\Gamma_{eq} = \underline{1}^T\Gamma\underline{1} \qquad ...(23)$$

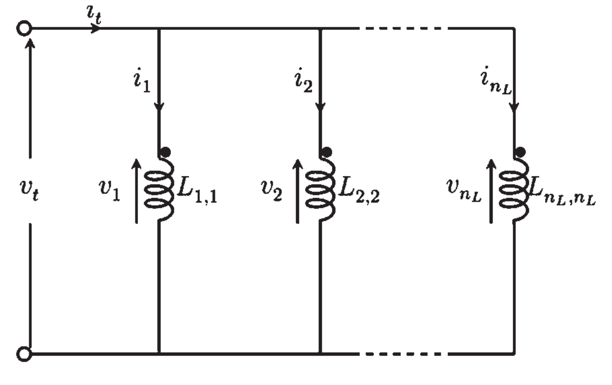

FIG. 6 PARALLEL CONNECTION OF n_L PROXIMALLY COUPLED INDUCTORS

The equivalent inductance L_{eq} of the parallel combination may then be written as

$$L_{eq} = \frac{1}{\Gamma_{eq}} = \frac{1}{\underline{1}^T\Gamma\underline{1}} \qquad ...(24)$$

Further, the current through the k_{th} inductor may be given as

$$\frac{i_k}{i_t} = \frac{\underline{1}^T\Gamma_k}{1^T\Gamma 1} = \frac{\underline{L}_k^T\underline{1}}{1^T \mathrm{L} 1} \qquad ...(25)$$

where $\underline{\Gamma}_k$ is the k^{th} column which is also the transpose of the k^{th} row) of T. The equations (23), (24), and (25) summarize the results of thisparallel connection. It may also be observed that the simple case of the uncoupled condition is now expressible as a special (and the most elementary) case of the above results.

2.8 Specific Particularization of the General Results to a Group of n_L Coupled R-L Branches

This specific configuration of the two-terminal network comprises of a series combination of a resistance R_k and an inductance $L_{k,k}$ such that

$$Z_{k,k} = R_k + L_{k,k}P \qquad ...(26)$$

and

$$Z_{k,j} = L_{k,j}\,P;\ k \neq j \qquad ...(27)$$

for all $k=1.2.\cdots\ n_L$. Thus the relevant adjusted primitive impedance matrix is

$$Z=R + L\,p \qquad ...(28)$$

where $R=diag[R_1,R_2,\cdots\ R_{nL}]$

and the admittance counterpart is

$$\mathrm{Y} = \mathrm{Z}^{-1} = [\mathrm{R+L\ p}]^{-1} \qquad ...(29)$$

2.8 Series Connection of n_L Coupled R-L Branches

Using (28) and applying the result (6) of the general case thereon, one readily obtains

$$Z_{eq} = 1^T R 1 + 1^T \mathrm{L} 1 p \qquad ...(30)$$

that is

$$Z_{eq} = R_{eq} + L_{eq}P \qquad ...(31)$$

as the equivalent impedance of the series combination of n_L coupled R-L branches, with R_{eq}= R and L_{eq} respectively as the equivalent resistance and inductance of this series combination. Here again, it may be observed that the simple case of the uncoupled condition is now expressible as a special (and the most elementary) case of the above results.

2.9 Parallel Connection of n_L Coupled R-L Branches

Using (29) and applying the result (12) of the general case thereon, one readily obtains

$$Y_{eq} = \underline{1}^T \mathrm{Y} \underline{1} = \underline{1}^T [\mathrm{R} + \mathrm{L}\,\mathrm{p}]^{-1} \underline{1} \qquad ...(32)$$

as the equivalent admittance (in operational form) of this parallel combination. This result appears as a rational function of the operator P even when the parameter matrices **R** and **L** have been number-specified. If it be desired to demonstrate the method with fully numerical results, a sinusoidal steady-state (for which, p≡jω), could be chosen as the operating state of the system, for this purpose of demonstration. Under such a state of affairs, the impedance matrix Z becomes complex-number-valued as in

$$\overline{\mathrm{Z}} = \mathrm{R} + j\omega \mathrm{L} = \mathrm{R} + j\mathrm{X} \qquad ...(33)$$

Thus the corresponding admittance matrix (also complex number-valued) takes the form

$$\overline{\mathrm{Y}} = [\overline{\mathrm{Z}}]^{-1} = [\mathrm{R} + j\omega\, \mathrm{L}\,]^{-1} \qquad ...(34)$$

Using (34) and applying the result (10) of the general case thereon, one obtains

$$\overline{Y}_{eq} = \underline{1}^T \overline{\mathrm{Y}} \underline{1} \qquad ...(35)$$

as the equivalent phasor-domain admittance of this parallel combination, and

$$\overline{Z}_{eq} = \frac{1}{\overline{Y}_{eq}} = \frac{1}{\underline{1}^T \overline{\mathrm{Y}} \underline{1}} \qquad ...(36)$$

as the equivalent phasor-domain impedance of the parallel combination of n_L coupled R-L branches.

3.0 NUMERICAL EXAMPLES – INDUCTOR NETWORKS

In this section examples involving proximally coupled inductors in series, parallel, and series-parallel combination will be considered. The results obtained in §2.6 and §2.7 will be put to use. The first two examples correspond to the elementary and ubiquitous case of two coupled inductors in series, and are mentioned here for the dual purpose of cataloguing and for demonstrating the use of the proposed method for well-known cases.The next two examples tackle two-inductor parallel combinations, which latter are less frequently encountered in literature. The remaining examples deal with combinations involving three or more inductors in series, parallel, and series-parallel, for which literary instances are almost non-existent.

3.1 Two-Inductor Combinations

The primitive arrangement of two coupled inductors is shown in Figure 7.

The primitive inductance matrix for this pair of inductors is given to be $\mathrm{L} = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 2 \end{bmatrix}$mH

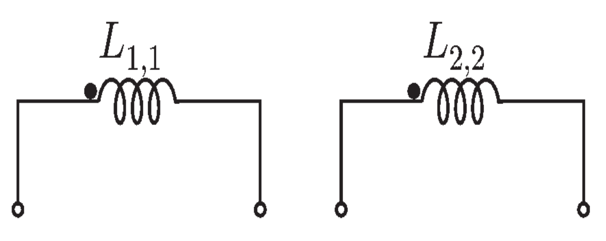

FIG. 7 PRIMITIVE DISPOSITION OF TWO COUPLED INDUCTORS (EXAMPLES 1-4)

3.1.1 Example 1: Two Inductors in Series – Ubiquitous Configuration 1

This connection is shown in Figure 8. This case corresponds to the first of the two ubiquitous examples to be found in literature.

In this case, the primitive inductance matrix needs no modification; thus, $\mathrm{L}=\mathrm{L}_p$.

The equivalent inductance of this series combination is given by

$$L_{eq} = \underline{1}^T \mathrm{L} \underline{1} = \boxed{4 \text{ mH}}$$

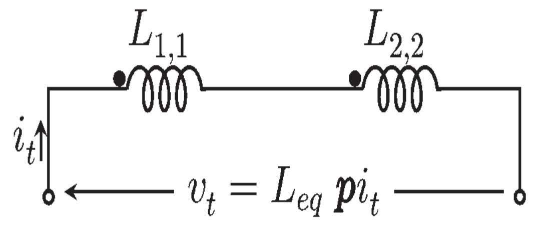

FIG. 8 EXAMPLE 1: TWO COUPLED INDUCTORS IN SERIES – UBIQUITOUS CONFIGURATION 1

3.1.2 Example 2: Two Inductors in Series – Ubiquitous Configuration 2

The connection is shown in Figure 9. This case corresponds to the second of the two ubiquitous examples to be found in literature.

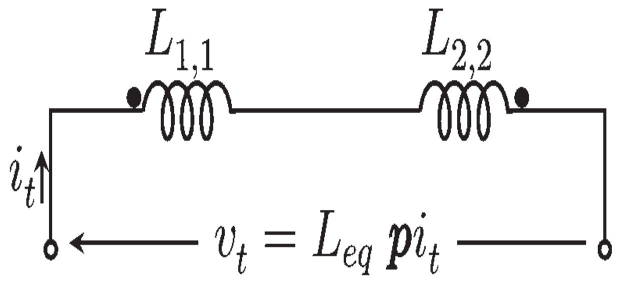

FIG. 9 EXAMPLE 2: TWO COUPLED INDUCTORS IN SERIES – UBIQUITOUS CONFIGURATION 2

The adjusted inductance matrix for this case is

$$\mathrm{L_p} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 2 \end{bmatrix} \mathrm{mH}$$

The equivalent inductance of this series combination is given by

$$L_{eq} = \underline{1}^T \mathrm{L} \underline{1} = \boxed{2\ \mathrm{mH}}$$

3.1.3 Example 3: Two Inductors in Parallel – Configuration 1

This connection is shown in Figure 10.

In this case, the primitive inductance matrix needs no modification; thus $\mathrm{L=L_p}$

The levitance matrix is therefrom obtained as

$$\Gamma = \mathrm{L}^{-1} = \left(\frac{1}{7}\right) \begin{bmatrix} 8 & -2 \\ -2 & 4 \end{bmatrix}$$

The equivalent levitance of the parallel combination is then given by

$$\Gamma_{eq} = \underline{1}^T \Gamma \underline{1} = \frac{8}{7}$$

whence,

$$L_{eq} = \frac{1}{\Gamma_{eq}} = \frac{7}{16} = \boxed{0.4375\ \mathrm{mH}}$$

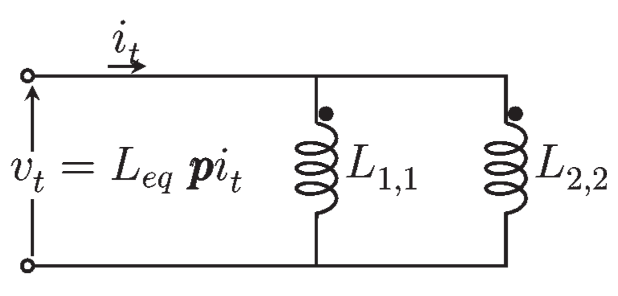

FIG. 10 EXAMPLE 3: TWO COUPLED INDUCTORS IN PARALLEL – CONFIGURATION 1

3.1.4 Example 4: Two Inductors in Parallel – Configuration 2

This connection is shown in Figure 11.

The adjusted inductance matrix for this case is

$$\mathrm{L} = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 2 \end{bmatrix} \mathrm{mH}$$

The levitance matrix is therefrom obtained as

$$\Gamma = \mathrm{L}^{-1} = \left(\frac{1}{7}\right) \begin{bmatrix} 8 & -2 \\ -2 & 4 \end{bmatrix}$$

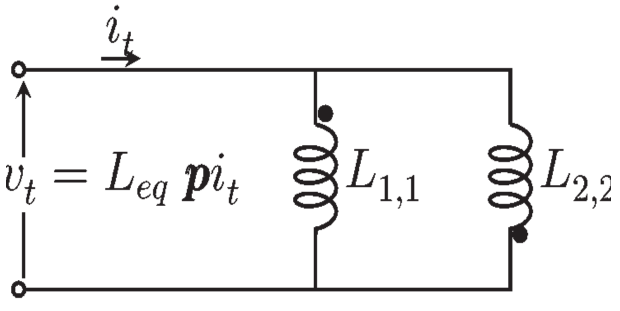

FIG. 11 EXAMPLE 4: TWO COUPLED INDUCTORS IN PARALLEL – CONFIGURATION 2

The equivalent levitance of the parallel combination is then given by

$$\Gamma_{eq} = \underline{1}^T \Gamma \underline{1} = \frac{16}{7}$$

whence,

$$L_{eq} = \frac{1}{\Gamma_{eq}} = \frac{7}{16} = \boxed{0.4375\ \mathrm{mH}}$$

3.2. Three-Inductor Combinations

The primitive arrangement of three coupled inductors is shown in Figure 12.

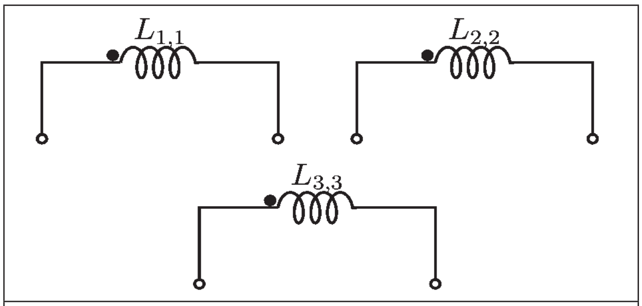

FIG. 12 PRIMITIVE DISPOSITION OF THREE COUPLED INDUCTORS (EXAMPLES 5-11)

The primitive inductance matrix for this group of inductors is reported to be

$$\mathrm{L_p} = \begin{bmatrix} 1 & 0.5 & 0.8 \\ 0.5 & 2 & 1 \\ 0.8 & 1 & 3 \end{bmatrix} \mathrm{mH}$$

3.2.1 Example 5: Three Inductors in Series – Configuration 1

This connection is shown in Figure 13.

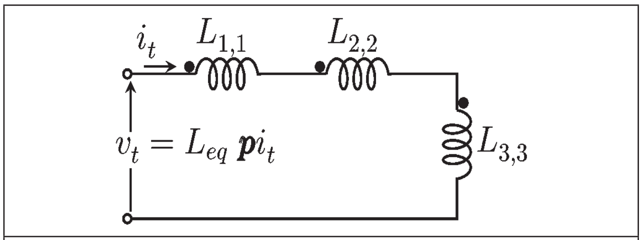

FIG.13 EXAMPLE 5: THREE COUPLED INDUCTORS IN SERIES – CONFIGURATION 1

In this case, the primitive inductance matrix needs no modification; thus L=L$_\mathrm{p}$

The equivalent inductance of this series combination is given by

$$L_{eq} = \underline{1}^T \mathrm{L} \underline{1} = \boxed{10.6\ \mathrm{mH}}$$

3.2.2 Example 6: Three Inductors in Series – Configuration 2

This connection is shown in Figure 14.

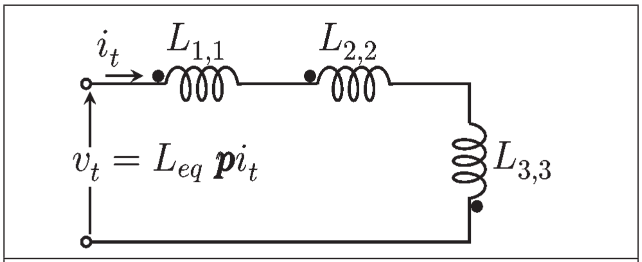

FIG. 14 EXAMPLE 6: THREE COUPLED INDUCTORS IN SERIES – CONFIGURATION 2

The adjusted inductance matrix for this case is

$$\mathrm{L} = \begin{bmatrix} 1 & 0.5 & -0.8 \\ 0.5 & 2 & -1 \\ -0.8 & -1 & 3 \end{bmatrix} \mathrm{mH}$$

The equivalent inductance of this series combination is given by

$$L_{eq} = \underline{1}^T \mathrm{L} \underline{1} = \boxed{3.4\ \mathrm{mH}}$$

3.2.3 Example 7: Three Inductors in Series – Configuration 3

This connection is shown in Figure 15.

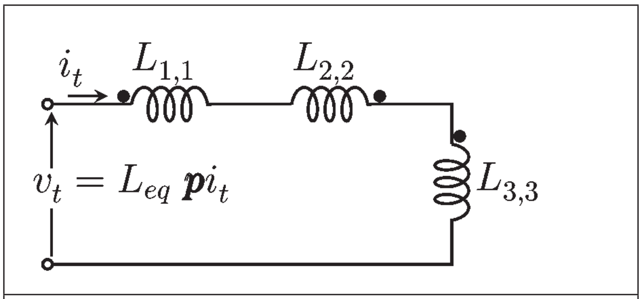

FIG. 15 EXAMPLE 7: THREE COUPLED INDUCTORS IN SERIES – CONFIGURATION 3

The adjusted inductance matrix for this case is

$$\mathrm{L} = \begin{bmatrix} 1 & -0.5 & 0.8 \\ -0.5 & 2 & -1 \\ 0.8 & -1 & 3 \end{bmatrix} \mathrm{mH}$$

The equivalent inductance of this series combination is given by

$$L_{eq} = \underline{1}^T \mathrm{L} \underline{1} = \boxed{4.6\ \mathrm{mH}}$$

3.2.4 Example 8: Three Inductors in Parallel– Configuration 1

This connection is shown in Figure 16.

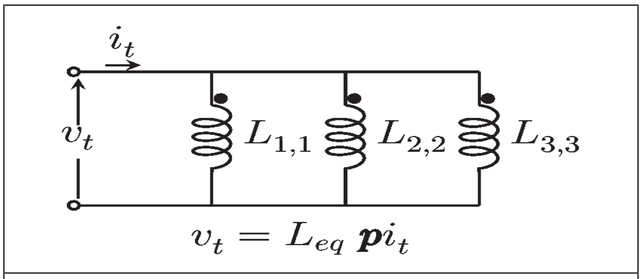

FIG. 16 EXAMPLE 8: THREE COUPLED INDUCTORS IN PARALLEL – CONFIGURATION 1

In this case, the primitive inductance matrix needs no modification; thus, $L=L_p$

The levitance matrix is there from obtained as

$$\Gamma = \left(\frac{1}{377}\right)\begin{bmatrix} 500 & -70 & -110 \\ -70 & 236 & -60 \\ 110 & -60 & 175 \end{bmatrix}$$

The equivalent levitance of the parallel combination is then given by

$$\Gamma_{eq} = \underline{1}^T \Gamma \underline{1} = \frac{431}{377}$$

whence,

$$L_{eq} = \frac{1}{\Gamma_{eq}} = \frac{377}{431} = \boxed{0.8747 \text{ mH}}$$

3.2.5 Example 9: Three Inductors in Parallel– Configuration 2

This connection is shown in Figure 17.

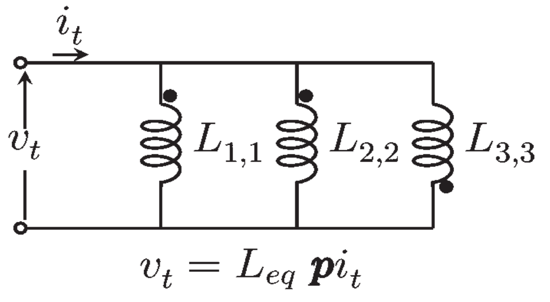

FIG. 17 EXAMPLE 9: THREE COUPLED INDUCTORS IN PARALLEL – CONFIGURATION 2

The adjusted inductance matrix for this case is

$$L = \begin{bmatrix} 1 & -0.5 & 0.8 \\ -0.5 & 2 & -1 \\ 0.8 & -1 & 3 \end{bmatrix} \text{mH}$$

The levitance matrix is therefrom obtained as

$$\Gamma = \left(\frac{1}{377}\right)\begin{bmatrix} 500 & -70 & -110 \\ -70 & 236 & -60 \\ 110 & -60 & 175 \end{bmatrix}$$

The equivalent levitance of the parallel combination is then given by

$$\Gamma_{eq} = \underline{1}^T \Gamma \underline{1} = \frac{1111}{377}$$

whence,

$$L_{eq} = \frac{1}{\Gamma_{eq}} = \frac{377}{1111} = \boxed{0.3393 \text{ mH}}$$

3.2.6 Example 10: Three Inductors in Parallel– Configuration 3

This connection is shown in Figure 18.

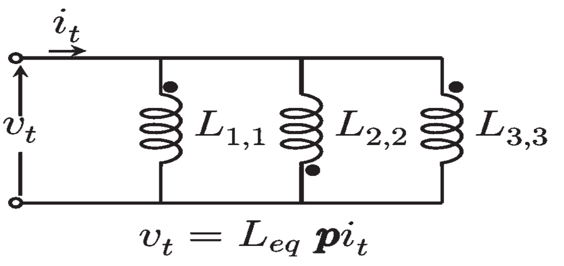

FIG. 18 EXAMPLE 10: THREE COUPLED INDUCTORS IN PARALLEL – CONFIGURATION 3

The adjusted inductance matrix is

$$L = \begin{bmatrix} 1 & -0.5 & 0.8 \\ -0.5 & 2 & -1 \\ 0.8 & -1 & 3 \end{bmatrix} \text{mH}$$

The levitance matrix is therefrom obtained as

$$\Gamma = \left(\frac{1}{377}\right)\begin{bmatrix} 500 & 70 & -110 \\ 70 & 236 & 60 \\ -110 & 60 & 175 \end{bmatrix}$$

The equivalent levitance of the parallel combination is then given by

$$\Gamma_{eq} = \underline{1}^T \Gamma \underline{1} = \frac{951}{377}$$

whence,

$$L_{eq} = \frac{1}{\Gamma_{eq}} = \frac{377}{951} = \boxed{0.3964 \text{ mH}}$$

3.2.7 Example 11: Three Inductors in a Mixed Series-Parallel Configuration

This connection is shown in Figure 19.

The adjusted inductance matrix for this case is

$$L = \begin{bmatrix} 1 & -0.5 & 0.8 \\ -0.5 & 2 & -1 \\ 0.8 & -1 & 3 \end{bmatrix} \text{mH}$$

The two inductors $L_{2,2}$ and $L_{3,3}$ in parallel must first be reduced to a single equivalent and then be joined in series to $L_{1,1}$.

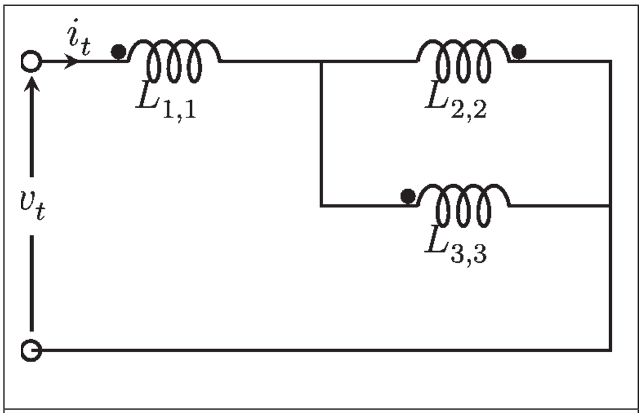

FIG. 19 EXAMPLE 11: THREE COUPLED INDUCTORS IN A MIXED SERIES/PARALLEL CONNECTION

The levitance matrix for the three inductors as a primitive group is first obtained as

$$\Gamma = \left(\frac{1}{377}\right)\begin{bmatrix} 500 & 70 & -110 \\ 70 & 236 & 60 \\ -110 & 60 & 175 \end{bmatrix}$$

This levitance matrix provides the means of performing the parallel reduction of the 2nd and 3rd inductors as follows: the 2x2 diagonal submatrix corresponding to the 2nd and 3rd inductors is reduced to a scalar via a simple summation of the elements thereof, followed by reducing the off-diagonal 1x2 submatrices being reduced likewise. This process results in a 2x2 matrix being formed by the retention of the original $T_{1,1}$ and the incorporationof the newly obtained reduced elements as

$$\boldsymbol{\Gamma}_{reduced} = \left(\frac{1}{377}\right)\begin{bmatrix} 500 & 70-110 \\ 70-110 & 236+60+60+175 \end{bmatrix}$$

That is

$$\boldsymbol{\Gamma}_{reduced} = \left(\frac{1}{377}\right)\begin{bmatrix} 500 & -40 \\ -40 & 531 \end{bmatrix}$$

This 2x2 matrix is now inverted to obtain a reduced inductance matrix to facilitate series reduction of the combination:

$$\mathbf{L}_{reduced} = [\Gamma_{reduced}]^{-1} = \left(\frac{1}{700}\right)\begin{bmatrix} 500 & 40 \\ 40 & 531 \end{bmatrix} \text{mH}$$

Whence, the equivalent inductance of this combination is

$$L_{eq} = \underline{\mathbf{1}}^T \mathbf{L}_{reduced} \underline{\mathbf{1}} = \boxed{1.5871 \text{ mH}}$$

3.3 Four-Inductor Combinations

The primitive arrangement of four coupled inductors is shown in Figure 20.

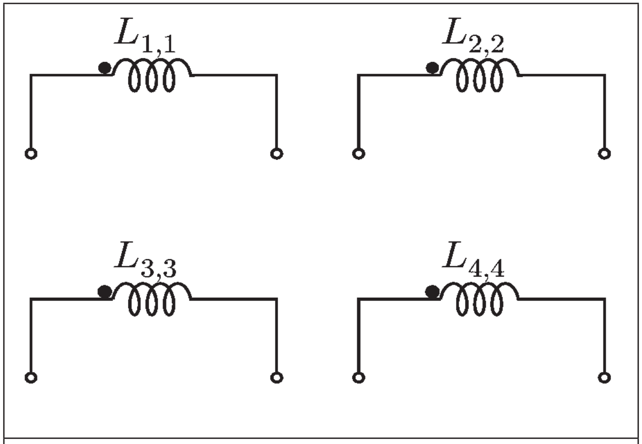

FIG. 20 PRIMITIVE DISPOSITION OF FOUR COUPLED INDUCTORS (EXAMPLES 12-17)

The primitive inductance matrix for this group of inductors is reported to be

$$L_P = \begin{bmatrix} 1 & 0.5 & 0.8 & 0.2 \\ 0.5 & 2 & 1 & 0.3 \\ 0.8 & 1 & 3 & 0.4 \\ 0.2 & 0.3 & 0.4 & 1.5 \end{bmatrix} \text{mH}$$

3.3.1 Example 12: Four Inductors in Series – Configuration 1

This connection is shown in Figure 21.

In this case, the primitive inductance matrix needs no modification; thus, $L = L_p$

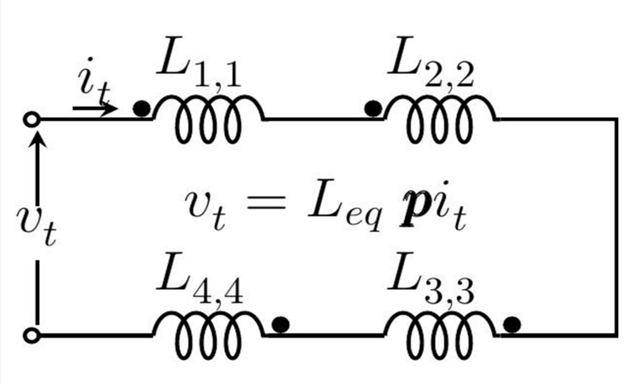

FIG. 21. EXAMPLE 12: FOUR COUPLED INDUCTORS IN SERIES – CONFIGURATION 1

The equivalent inductance of this series combination is given by

$$L_{eq} = \underline{1}^T \mathrm{L} \underline{1} = \boxed{13.9\ \mathrm{mH}}$$

3.3.2 Example 13: Four Inductors in Series – Configuration 2

This connection is shown in Figure 21.

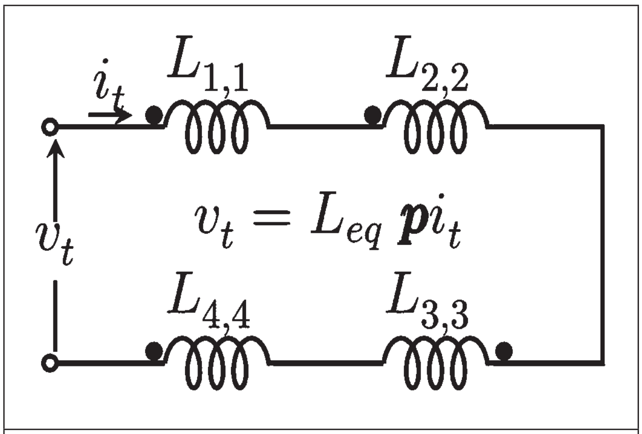

FIG. 22 EXAMPLE 13: FOUR COUPLED INDUCTORS IN SERIES – CONFIGURATION 2

The adjusted inductance matrix for this case is

$$\mathrm{L} = \begin{bmatrix} 1 & 0.5 & 0.8 & -0.2 \\ 0.5 & 2 & 1 & -0.3 \\ 0.8 & 1 & 3 & -0.4 \\ -0.2 & -0.3 & -0.4 & 1.5 \end{bmatrix} \mathrm{mH}$$

The equivalent inductance of this series combination is given by

$$L_{eq} = \underline{1}^T \mathrm{L} \underline{1} = \boxed{10.3\ \mathrm{mH}}$$

3.3.3 Example 14: Four Inductors in Series – Configuration 3

This connection is shown in Figure 23.

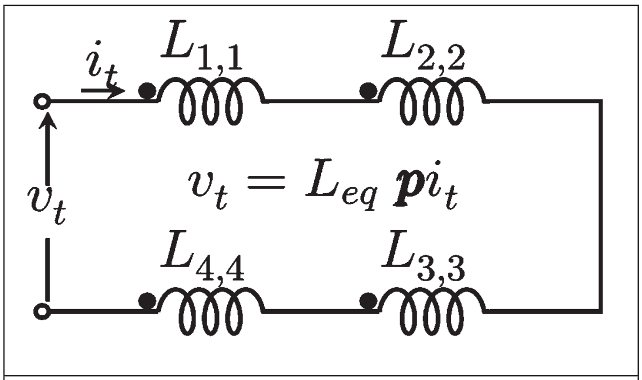

FIG. 23 EXAMPLE 14: FOUR COUPLED INDUCTORS IN SERIES – CONFIGURATION 3

The adjusted inductance matrix for this case is

$$\mathbf{L} = \begin{bmatrix} 1 & 0.5 & -0.8 & -0.2 \\ 0.5 & 2 & -1 & -0.3 \\ -0.8 & -1 & 3 & 0.4 \\ -0.2 & -0.3 & 0.4 & 1.5 \end{bmatrix} \mathrm{mH}$$

The equivalent inductance of this series combination is given by

$$L_{eq} = \underline{1}^T \mathrm{L} \underline{1} = \boxed{4.7\ \mathrm{mH}}$$

3.3.4 Example 15: Four Inductors in Parallel – Configuration 1

This connection is shown in Figure 24.

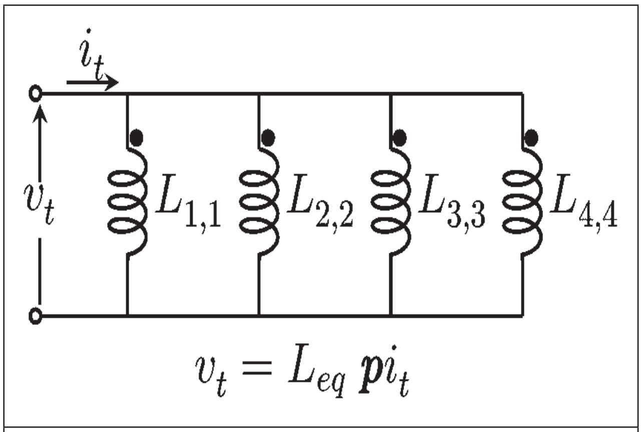

FIG. 24 EXAMPLE 15: FOUR COUPLED INDUCTORS IN PARALLEL – CONFIGURATION 1

In this case, the primitive inductance matrix needs no modification; thus, $L = L_p$

The levitance matrix is therefrom obtained as

$$\mathbf{\Gamma} = \begin{bmatrix} 1.33 & -0.18 & -0.29 & -0.07 \\ -0.18 & 0.63 & -0.15 & -0.06 \\ -0.29 & -0.15 & 0.49 & -0.06 \\ -0.07 & -0.06 & -0.06 & 0.70 \end{bmatrix}$$

The equivalent levitance of the parallel combination is then given by

$$\Gamma_{eq} = \underline{1}^T \Gamma \underline{1} = 1.53$$

whence,

$$L_{eq} = \frac{1}{\Gamma_{eq}} = \frac{1}{1.53} = \boxed{0.6542\ \mathrm{mH}}$$

3.3.5 Example 16: Four Inductors in Parallel – Configuration 2

This connection is shown in Figure 25.

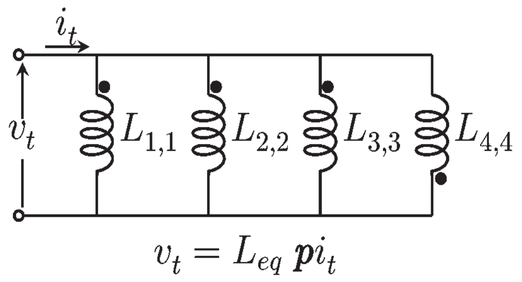

FIG. 25 EXAMPLE 16: FOUR COUPLED INDUCTORS IN PARALLEL – CONFIGURATION 2

The adjusted inductance matrix for this case is

$$L = \begin{bmatrix} 1 & 0.5 & 0.8 & -0.2 \\ 0.5 & 2 & 1 & -0.3 \\ 0.8 & 1 & 3 & -0.4 \\ -0.2 & -0.3 & -0.4 & 1.5 \end{bmatrix} \text{mH}$$

The levitance matrix is therefrom obtained as

$$\Gamma = \begin{bmatrix} 1.33 & -0.18 & -0.29 & 0.07 \\ -0.18 & 0.63 & -0.15 & 0.06 \\ -0.29 & -0.15 & 0.49 & 0.06 \\ 0.07 & 0.06 & 0.06 & 0.70 \end{bmatrix}$$

The equivalent levitance of the parallel combination is then given by

$$\Gamma_{eq} = \underline{1}^T \Gamma \underline{1} = 2.26$$

whence,

$$L_{eq} = \frac{1}{\Gamma_{eq}} = \frac{1}{2.26} = \boxed{0.443 \text{ mH}}$$

3.3.6 Example 17: Four Inductors in Parallel – Configuration 3

This connection is shown in Figure 26. The adjusted inductance matrix for this case is

$$L = \begin{bmatrix} 1 & 0.5 & -0.8 & -0.2 \\ 0.5 & 2 & -1 & -0.3 \\ -0.8 & -1 & 3 & 0.4 \\ -0.2 & -0.3 & 0.4 & 1.5 \end{bmatrix} \text{mH}$$

The levitance matrix is therefrom obtained as

$$\Gamma = \begin{bmatrix} 1.33 & -0.18 & -0.29 & 0.07 \\ -0.18 & 0.63 & 0.15 & 0.06 \\ -0.29 & 0.15 & 0.49 & -0.06 \\ 0.07 & 0.06 & -0.06 & 0.70 \end{bmatrix}$$

The equivalent levitance of the parallel combination is then given by

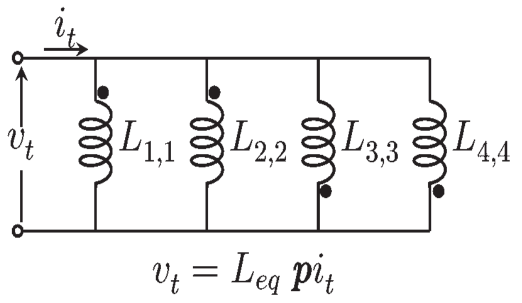

FIG. 26 EXAMPLE 17: FOUR COUPLED INDUCTORS IN PARALLEL – CONFIGURATION 3

$$\Gamma_{eq} = \underline{1}^T \Gamma \underline{1} = 3.79$$

whence,

$$L_{eq} = \frac{1}{\Gamma_{eq}} = \frac{1}{3.79} = \boxed{0.2633 \text{ mH}}$$

4.0 NUMERICAL EXAMPLES – R-L NETWORKS

In this section examples involving proximally coupled R-L branches in parallel combination will be considered. The results obtained in §2.8 will be put to use. The computation of the equivalent phasor-domain impedance will be demonstrated. The treatment of the series combination of such branches has already been discussed in its entire generality in §2.8.1, and, as shown therein, happens to be merely an extension of that for the series combination of inductors – thus occasioningno additional attention hereafter.

4.1 Example 18: Parallel Combination of two R-L Branches

The primitive disposition of two proximally coupled R-L branches is shown in Figure 27.

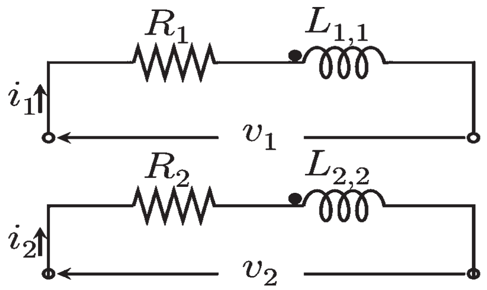

FIG. 27 PRIMITIVE DISPOSITION OF TWO COUPLED R-L BRANCHES

The phasor-domain primitive impedance matrix of this pair of R-L branches, evaluated at a 50 Hz (ω=100π rad/s) sinusoidal steady-state is reported to be

$$\bar{Z}_P = \begin{bmatrix} 0.1 + j0.1\pi & j0.05\pi \\ j0.05\pi & 0.4 + j0.2\pi \end{bmatrix} \Omega$$

$$\bar{Z}_P = R_P + j\omega L_P = R_P + jX_P$$

where $R_P = \mathrm{diag}[0.1, 0.4]\ \Omega$
Consider next the parallel combination of these two R-L branches as shown in Figure 28.

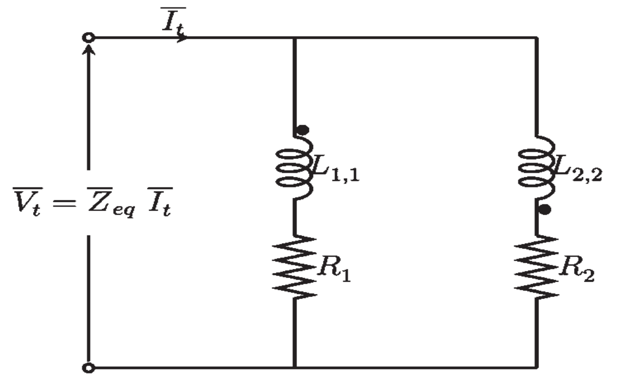

FIG. 28 EXAMPLE 18: TWO COUPLED R-L BRANCHES IN PARALLEL (PHASOR-DOMAIN $\bar{Z}_{eq}$)

The adjusted impedance matrix for this case is

$$\bar{Z} = \begin{bmatrix} 0.1 + j0.1\pi & -j0.05\pi \\ -j0.05\pi & 0.4 + j0.2\pi \end{bmatrix} \Omega$$

The admittance matrix is therefrom obtained as $\bar{\mathbf{Y}} = [\bar{\mathbf{Z}}]^{-1}$. The equivalent phasor-domain admittance of the parallel combination is then obtained as

$$\bar{Y}_{eq} = \underline{\mathbf{1}}^T \bar{\mathbf{Y}} \underline{\mathbf{1}} = (3.2084 - j4.9116)\ \mathrm{S}$$

whence, the equivalent phasor-domain impedance of this parallel combination is

$$\bar{Z}_{eq} = \frac{1}{\bar{Y}_{eq}} = \boxed{(0.0932 + j0.1427)\ \Omega}$$

4.2 Example 19: Parallel Combination of three R-L Branches

The primitive disposition of three proximally coupled R-L branches is shown in Figure 29.

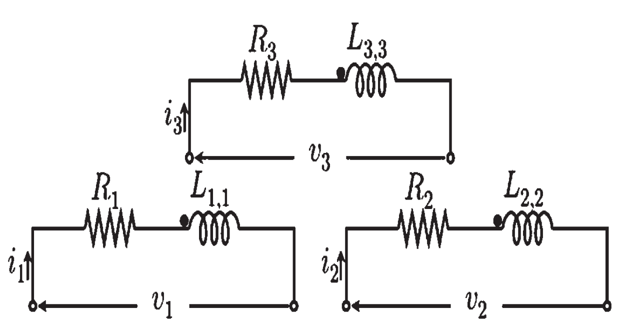

FIG. 29 PRIMITIVE DISPOSITION OF THREE COUPLED R-L BRANCHES

The phasor-domain primitive impedance matrix of this group of R-L branches, evaluated at a 50 Hz (ω=100π rad/s) sinusoidal steady-state is reported to be

$\bar{Z}p = R = jXp$

where R = diag $[0.1 \quad 0.4 \quad 0.5]\ \Omega$ and

$$\mathbf{X_P} = \pi \begin{bmatrix} 0.1 & 0.05 & 0.08 \\ 0.05 & 0.2 & 0.1 \\ 0.08 & 0.1 & 0.3 \end{bmatrix} \Omega$$

Consider next the parallel combination of these three R-L branches as shown in Figure 30.

The adjusted impedance matrix for this case is

$\bar{Z} = R + j\,X$

where R = diag $[0.1 \quad 0.4 \quad 0.5]\ \Omega$, and

$$X = \pi \begin{bmatrix} 0.1 & -0.05 & 0.08 \\ -0.05 & 0.2 & -0.1 \\ 0.08 & -0.1 & 0.3 \end{bmatrix} \Omega$$

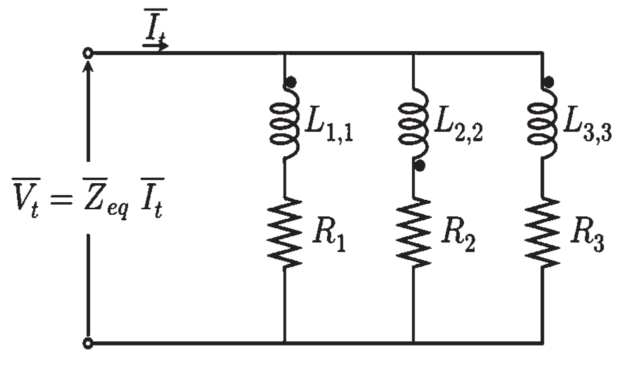

FIG. 30 EXAMPLE 19: THREE COUPLED R-L BRANCHES IN PARALLEL (PHASOR-DOMAIN $\bar{Z}_{eq}$)

The admittance matrix is therefrom obtained as $\overline{\mathbf{Y}} = [\overline{\mathbf{Z}}]^{-1}$ The equivalent phasor-domain admittance of the parallel combination is then obtained as

$$\overline{Y}_{eq} = \underline{\mathbf{1}}^T \overline{\mathbf{Y}} \underline{\mathbf{1}} = (3.4838 - j5.2537)\ \mathrm{S}$$

whence, the equivalent phasor-domain impedance of this parallel combination is

$$\overline{Z}_{eq} = \frac{1}{\overline{Y}_{eq}} = \boxed{(0.0877 + j0.1322)\ \Omega}$$

4.3 Example 20: Parallel Combination of Four R-L Branches

The primitive disposition of four proximally coupled R-L branches is shown in Figure 31.

The phasor-domain primitive impedance matrix of this group of R-L branches, evaluated at a 50 Hz (ω=100π rad/s) sinusoidal steady-state is reported to be

$\overline{Z}_P = R + jX_p$

where R = diag [0.1 0.4 0.5 0.2] Ω, and

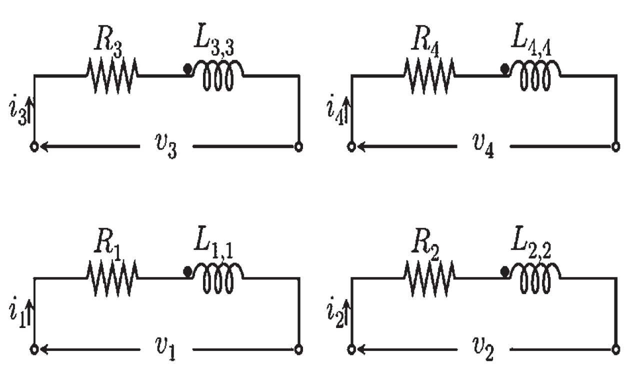

FIG. 31 PRIMITIVE DISPOSITION OF FOUR COUPLED R-L BRANCHES

$$X_P = \pi \begin{bmatrix} 0.1 & 0.05 & 0.08 & 0.02 \\ 0.05 & 0.2 & 0.1 & 0.03 \\ 0.08 & 0.1 & 0.3 & 0.04 \\ 0.02 & 0.03 & 0.04 & 0.15 \end{bmatrix} \Omega$$

Consider next the parallel combination of these four R-L branches as shown in Figure 32.

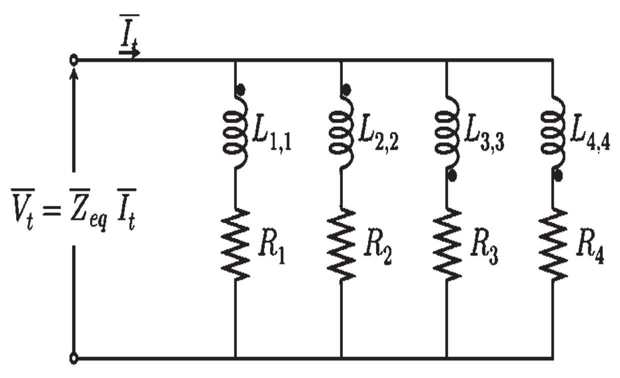

FIG. 32 EXAMPLE 20: FOUR COUPLED R-L BRANCHES IN PARALLEL (PHASOR-DOMAIN $\overline{Z}_{eq}$)

The adjusted impedance matrix for this case is $\overline{Z}$= R+ j X where R = diag [0.1 0.4 0.5 0.2] Ω , and

$$\frac{\mathbf{X}}{\pi} = \begin{bmatrix} 0.1 & 0.05 & -0.08 & -0.02 \\ 0.05 & 0.2 & -0.1 & -0.03 \\ -0.08 & -0.1 & 0.3 & 0.04 \\ -0.02 & -0.03 & 0.04 & 0.15 \end{bmatrix} \Omega$$

The admittance matrix is therefrom obtained as $\overline{\mathbf{Y}} = [\overline{\mathbf{Z}}]^{-1}$ The equivalent phasor-domain admittance of the parallel combination is then obtained as

$$\overline{Y}_{eq} = \underline{\mathbf{1}}^T \overline{\mathbf{Y}} \underline{\mathbf{1}} = (4.9098 - \mathrm{j}8.2259)\ \mathrm{S}$$

whence, the equivalent phasor-domain impedance of this parallel combination is

$$\overline{Z}_{eq} = \frac{1}{\overline{Y}_{eq}} = \boxed{(0.0535 + \mathrm{j}0.0896)\ \Omega}$$

5.0 CONCLUSIONS

A simple, general, elegant, and scalable method for obtaining of the equivalent impedance (or admittance) of series and parallel configurations of proximally coupled two-terminal networks has been presented. This method has been amply illustrated by twenty examples of series, parallel, and series-parallel configurations of inductors and inductor-resistor combinations. The method is based on the usage of the primitive impedance (or admittance) matrices and is easily generalizable to networks of any size. The task of obtaining the series (or parallel) equivalent impedance (or admittance) is thus reduced, generalized, and simplified to a straightforward process of addition of the elements of the appropriate primitive matrix. The presented method has been demonstrated to accommodate, as a special (and most elementary) instance, the case of the reduction of networks sans proximal coupling. The elegance and scalability of the presented method stems almost entirely from the vector-matrix formulation of the performance equations of the primitive network and by retention of the time-domain in these equations facilitated by the Heaviside differential operator.

This method could be suitably adapted with minimal modifications to tackle more general block-level topologies as well as to other forms of reduction. A companion paper, which iscurrently in manuscript preparation stage, proposes to deal more comprehensively with the above details.

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