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Evolutionary approach to stackelberg game based demand response for electric vehicle charging

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Abstract: In this era of industrialization and modernization, global warming and climate change are pressing issues. In order to address that transportation electrification seems to be a potential solution owing to much lower carbon and nitrogen oxides emissions as opposed to their Internal Combustion Engine (ICE) vehicles counterparts. But to realize this customers need to be motivated to adopt this new technology. There should be opportunity for customers to exploit the elastic nature of EV load by participating in the electricity market and adjust their consumption levels such that they derive maximum satisfaction with the price at which they buy the electricity. This introduces the concept of Demand response, which aims at reduction of power generation costs and electricity bills by allowing con- trol of electricity consumption through electricity prices. This interaction between retailer and customers leads to a conflict of interest as every entity aims at maximizing their benefits. In this study, a Stackelberg game model has been developed where the retailer sets electricity price with the knowledge of EV customers' behavior so as to maximize its profit and the EV customers set consumption level to maximize their payoffs. We have proven that game settles down at an equilibrium point where EV charging requirements are met.

1.0 INTRODUCTION

About 15 percent of the man-made C O2 in the entire world comes from cars, airplanes, trucks, ships and other internal combustion engine (ICE) vehicles. In the U.S., the transportation sector is responsible for almost one third of their climate-changing emissions, owing to their near-total dependence on petroleum fuels [1]. In order to combat global warming and climate change, reducing emissions coming from transportation sector is quintessential. In the wake of realizing this, transportation electrification seems a potential solution owing to much lower carbon and nitrogen oxides emissions as opposed to their Internal Combustion Engine (ICE) vehicles counterparts. But to realize this customers need to be motivated to adopt this new technology. They should be availed electricity for Electric Vehicles (EVs) charging at cheaper prices. This calls in for a need to increase the involvement of EV users in the electricity market. But in spite of having several advantages, due to the lack of coordination in the charging of EVs this large scale deployment of EVs will certainly pose a threat to the normal operations of power system. The negative impacts these newly added EVs will have on the power system are visible from the demonstrations of several studies [2]. Demand patterns that require line flows to exceed specified limits lead to grid congestion. To allow increased penetration of EVs without vio- lation of specified limits and constraints, coordinated charging of EVs is an effective solution. The proposed objectives for charging coordination include EV penetration maximization [3], and minimization of customer charging costs [4], [5] and EV charging co-ordination is possible in

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multiple ways if the power grid has provisions for it.

Transition of the existing power grid to smart grid (SG) will enable the demand seeking entity to play a significant role. There are several challenges like price volatility, congestion, capacity shortages, energy and fuel efficiencies and greenhouse gas emissions, faced by the existing grid which need to be addressed. Hence the need for a reliable, environmentally friendly, affordable, and consumer interactive power makes the large scale deployment of enabling technologies justifiable, in order to ensure transition of the current grid into a smarter grid. Smart grid utilizes a real-time two-way communication system which has the capability to control the electricity demand in an adaptive manner. By enabling controllability of the electricity demand, the demand at peak hours can be shifted to off-peak hours thus reducing the generation costs. The smart grid places key emphasis on the role of consumers in electricity market operations [6].

Demand Response (DR), an essential key feature of the smart grid, can be defined as change in consumption of electricity by the customers in response to the changes made in the price of electricity by the Utility Company or retailer over a period of time. There are several DR strategies, such as 1) Rate or price based DR programs; 2) Incentive-based DR programs and 3) Demand reduction bids [7]. Demand response is an effective way of controlling demand as it applies different rates at different hours throughout the day. By this program, the users can considerably reduce their electricity bills by shifting the consumption of their elastic loads from peak hours to off peak hours.

As EV load is deferrable by nature and can be seen as an increased load on the system, the large scale adoption of EVs make it essential for the utilities to avail adequate DR programs for EV users. The elastic nature of EV load enables the customers to shift their load from peak to off peak period, reducing energy costs, overall cost of generation by smoothening the load curve, market prices of electricity hence preventing the generating companies from exerting market power. In our work we focus on the rate or price-based DR programs. In this type of DR program, DR is implemented through approved utility tariffs in deregulated markets according to which the price of electricity varies with time so as to motivate customers to shift their consumption.

Recent studies on DR have focused majorly on the two areas: retailer oriented and customer oriented. Considerable amount of work has been done on supply-demand balancing and market clearance in power systems [8]. At the planning and generation level, these studies have focused on the eco- nomic aspects but have not considered user payoff as an important component. Work on the user-utility attempts to maximize their utilities, without considering the generation cost or revenue of the retailer or UCs. This fact has motivated us to address the payoff maximization of EV users along with the revenue maximization of the retailer. We aim to bridge the gap between the two existing research directions through this work. Some work has been done in the past to design games for demand response [9]- [10]. Energy consumption games to offer an incentive to the users who cooperate, are formulated in [9] and [11]. However, the games are designed considering users as the only players. Stackelberg games which take place between retailers and users were proposed in [12] and [13], once the retailer announces its electricity price in order to maximize its revenue, each user decides its consumption accordingly to maximize its utility. However, this game doesn't consider the constraint of charging requirements of EVs. In [10], an optimal ToU pricing method using game theory has been introduced without considering user constraints. In [14], a demand response problem has been proposed as a game between a retailer and customers considering the constraints of EV charging at home.

To the best of our knowledge, much work has not been done to capture the interactions between EVs and the grid through distributed models and algorithms. Thus in this work we develop a model that captures the conflicting objectives between the retailer, who seeks to maximize its profit by max- imizing the revenue, and the EVs users who seek to minimize their charging cost. We have modelled the demand response problem as a Stackelberg game between retailer and customers considering the constraint of EV charging requirements in various scenarios with different types of customers. In our game, the retailer is the leader and customers are the followers. The retailer aims to maximize its profit by setting electricity price such that the charging requirements of each customer are satisfied. Retailer has a prior knowledge of customers' behavior, as to how they will react to a particular price set by the retailer, which helps it to strategize accordingly. Each customer decides consumption of electricity according to the price declared by the retailer at any hour. A utility function to reflect the demand and meet the charging requirements of each customer has been given and the existence of equilibrium in our proposed game is proven.

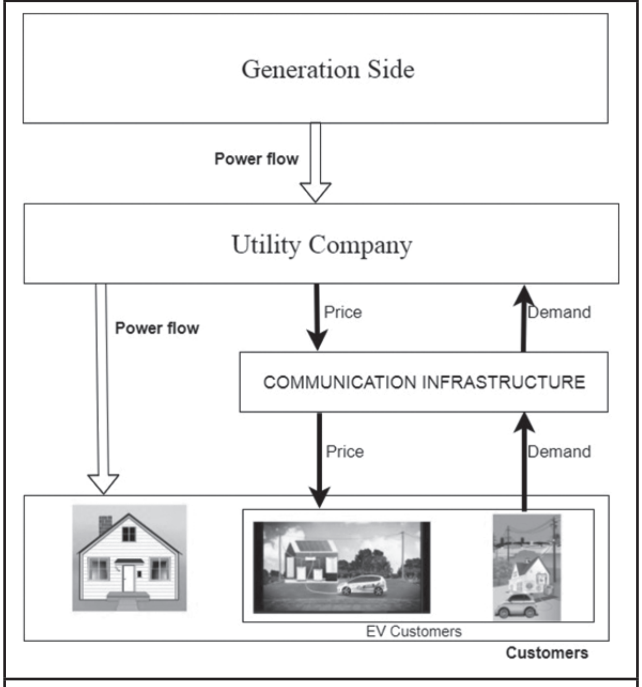

FIG. 1. MODEL INDICATING COMMUNICATION BETWEEN RETAILER AND MULTIPLE USERS [14], [15], [16]

2.0 MODELLING OF THE SYSTEM

We consider N number of customers and one retailer who is responsible to provide electricity to the customers. As shown in fig 1, there exists three layers in our model namely generators, retailer and the end-users also known as customers, which comprise generation, distribution and consumption. The retailer acquires power from the generators which is a separate process, which is beyond the scope of this paper. We emphasize on the interaction between the retailer and the customers. The customers are said to receive information of prices from time-to-time from the retailer and they respond to it in terms of demand. This communication involving data is performed via communication channels which use WiFi, WiMAX, LTE etc. The generation could be either from renewable or non- renewable energy sources. Though the generators working on fossil fuels have power available all the time, which in a way assures reliable supply but they give rise to pollution. Whereas, the renewable energy in spite of being free of pollution, faces challenge due to its inherent intermittency. We however suggest a huge chunk of electricity coming from renewable energy sources otherwise even after switching to EVs there is a possibility of shift in the source of pollution.

The retailer aims to maximize his profit by reselling the electricity to customers that he buys in the day ahead market. On division of each day into T time periods, period index is given by t. In order to charge EVs, the customers will have to buy electricity from retailer and our focus in this paper is on this particular transaction. A contract is signed by the retailer to provide electricity to its customers for the charging of EVs.

So the retailer is obliged to fulfill the charging requirements of EVs. Every hour the retailer declares the price of electricity pt and as a response to this price, every customer decides its consumption level xt in kW. We have proposed a leader- follower game which is Stackelberg game in which the retailer who is the leader, adopts the strategy based on his knowledge of how the consumer will react even before the customers who are the followers, take any decision.

A. Demand modelling of customers

The customers in our model are assumed to have both elastic and non-elastic loads. There will be some loads which cannot be controlled based on prices, for example refrigerators, lights etc. These loads fall in the category of non-elastic loads which will not respond to the change in prices on a timely basis. But when it comes to EVs, they need to be charged for some time out of the total time they are plugged in to the grid. This fact makes them elastic loads as far as their energy requirements are met. The EV load can be modelled as:

$$\sum_{t \in C_i} \mu_c x_i^t = E_i$$
$$0 \leq x_i^t \leq \delta_i, if\ t \in C_i$$
$$x_i^t = 0, if\ t \notin C_i \qquad ...(1)$$

Where μc , Ei and δ represent charging efficiency of battery, energy requirement (in kW) of an EV to get fully charged, and rate of maximum charging for customer i in kW respectively. The charging period interval is denoted by *Ci* and it is given as *Ci* = [*Si* , *Fi*], where Si indicates the start of charging and *Fi* denotes the end of charging.

A utility function as a function of x is defined in order to express degree of satisfaction on consumption of x units of electricity in some given time. The utility function considered by us is non-decreasing, there is a decrease in marginal satisfaction with an increase in consumption, limit is imposed on it and on same amount of consumption, the customer with higher weight derives higher satisfaction. We have considered the utility function as taken in [17], it is a quadratic function where marginal satisfaction decreases linearly. It is given by:

$$U_i(x_i, w_i, \delta_i) = \begin{cases} w_i x_i - \frac{w_i}{2\delta_i} x_i^2, & if\ 0 \leq x_i \leq \delta_i \\ \frac{w_i \delta_i}{2}, & if\ x_i \geq \delta_i \end{cases} \qquad (2)$$

where $\mathbf{x_i}$ is the consumption of electricity in kilowatts and $\mathrm{w_i}$ is the weight of customer i. Here it can be seen that the maximum utility occurs at $\mathrm{x} = \delta_\mathbf{i}$ and it doesn't increase beyond that value.

B. Cost modelling of retailers

The cost at which electricity is bought by the retailer from the generators varies from time-to-time. As seen from the recent research [18], how to model the cost function with reference to the amount of electricity is a hot area of research but beyond the scope of our paper. An assumption about the cost function is that it is differentiable, increasing and convex function with respect to x.

3.0 DESIGN OF STACKELBERG GAME

A natural paradigm to model the behaviors of the retailer and users along with their interactions is provided by Game theory. We propose a Stackelberg game with one retailer and n-customers. Initially, the retailer decides upon a set of prices in order to maximize its profit. The retailer comes up with these prices based on his knowledge of customers' behaviors and he informs customers about it. Then the customers respond to these prices by adjusting an optimal consumption of electricity. Since the retailer acts first followed by the customers, who make their decisions based on the prices set by the retailer, these are sequential events with a leader and many followers. Initially, we analyze the customers' behaviors later we do the same for the retailer as well, hence we adopt a backward induction technique for reaching at an equilibrium point of the Stackelberg game [14]. The optimal solution forms the equilibrium of the game.

A. Customer Side Analysis

Each consumer will decide it's consumption level according to the prices to maximize it's benefits. The payoff function of customer is given below:

$$g_i(x) = U_i(x_i, w_i, \delta_i) - p x_i \qquad ...(3)$$

The first term represents the utility that customer i derives on consumption of xi with weight wi . The second term is expenditure of customer i. The payoff function can be said to be differentiable because the utility function is differentiable and the expenditure term is linear.

The first derivative of payoff is

$$\frac{dg_i}{dx_i} = \begin{cases} w_i - p - \frac{w_i}{\delta_i} x_i, & if\ 0 \leq x_i \leq \delta_i \\ -p, & if\ x_i \geq \delta_i \end{cases} \qquad ...(4)$$

Using the condition of stationary point for the payoff function $\frac{dg_i(x)}{dx_i}$= 0, we arrive at best consumption response for a price p:

$$x_i^*(p) = \begin{cases} \delta_i(1 - \frac{p}{w_i}), & p \leq w_i \\ 0, & otherwise \end{cases} \quad ...(5)$$

It can be seen from (5) that the optimal consumption x_i^* of each customer i depends on the maximum charging rate (δi), declared price (*p*) and weight (w_i).

B. Retailer Side Analysis

An assumption is that before the retailer sets the price, he is aware of each customer's requirements, like Ei , Ci , δi and wi . The retailer decides the price pi so as to maximize his profit by speculating that x_i^* (*p*) is consumption by consumer i. If P is the final decision vector, the retailer sets for its consumer at each hour, we can find the hourly payoff of retailer as follows:

$$f(p^t) = R(p^t) - C(X^t) \quad ...(6)$$

where R(.) is the revenue generated as a function of price p, and C(.) is the cost of generating the electricity as a function of demand X. The revenue is basically price multiplied by total consumption of all customers, mathematically it is represented as $p^t \sum_{i \in N} x_i^t$. We use a quadratic cost function C(X_t) = a(X_t)^2, from [19], where X^t is the total load in hour h. The cost function is a concave, increasing, and differentiable function, it includes price-elastic load of each customer also the consumption by residential and commercial base loads. It is given by $X^t = x_o^t + \sum_{i \in N} x_i^t$, x_o^t is the total residential and commercial base loads at t. Optimization Problem for retailer is as follows:

$$\textbf{(P)} \; max \sum_{t \in C} f(p^t)$$

subject to:

$$\sum_{t \in C} \mu_c x_i^t = E_i, \; for \; all \; i \in N$$

$$p^t \geq 0 \; for \; all \; t \in C$$

In order to assure a feasible solution, the charging requirement of each customer should be less than that amount when for the entire charging interval, the customer would charge at the maximum charging rate. It is mathematically represented as: $\delta_i |C_i| \geq E_i/\mu_c$

1) Single-customer case: Now we illustrate the problem for a single customer case here. Consider the optimization problem (P), we try to maximize retailer's payoff when a single customer is involved. By using x_i^* from (4) in (P), we can formulate the optimization problem as follows:

$$\textbf{(P1)} \; max \sum_{t \in C} (p^t x_i^t - a(x_0^t + x_i^t)^2) \quad ...(7)$$

subject to:

$$\sum_{t \in C_i} \mu_c x_i^t = E_i \quad ...(8)$$

$$p^t \geq 0 \; for \; all \; t \in C_i \quad ...(9)$$

In (5), since x/(p) is zero for price value greater than the weight of customer, optimal solution will not exist for any price more than the weight. So, we consider only the prices less than the weight, i.e. $0 \leq p^h \leq w_i$. In that case, substituting $x_i^* = \delta_i(1 - p^h/w_i)$ in (P1), we get:

$$\sum_{t \in C_i} \left(-\left(\frac{\delta_i}{w_i} + a\frac{\delta_i^2}{w_i^2} \right) (p^t)^2 + \left(\delta_i + 2a\frac{\delta_i^2}{w_i^2}(1 + x_o^t) \right) p^t - a\delta_i^2(1 + x_o^t)^2 \right) \quad ...(10)$$

The constraint becomes:

$$\sum_{t \in C_i} p^t = w \left(C_i - \frac{E_i}{\mu_c \delta_i} \right) \quad ...(11)$$

It can be seen from (10) that the objective function has become negative quadratic concave function and from (11), we can say that the constraints are linear. Hence, (P1) is a

problem of convex optimisation. We find p*using Particle Swarm Optimization(PSO) as explained in Section V. For the sake of simplicity, we are giving the mathematical formulation of the case when for all $t \in C_i$ all base loads are zero. For this case, the solution is:

$$p^{t*} = w_i \left(1 - \frac{E_i}{\mu_c \delta_i C_i}\right) \; for \; all \; t \in C_i \quad ...(12)$$

and the consumption by each customer is given by:

$$x^{h*}_{i,g} = \frac{E_i}{\mu_c C_i} \; for \; all \; t \in C_i \quad ...(13)$$

2) Multiple-customers' case: Using the previous results for single-customer case, we extend it for multiple-customers case. As the optimization problem P isn't convex, by adding an assumption, we make it convex. We have derived the solution for $0 \leq p^t \leq w_i$ in case of single customer, hence when it comes to multiple customers, for the above results to hold good, we add the following constraint:

$$p^t \leq min(w_n : for \; all \; n \in N), for \; all \; t \in C \quad ...(14)$$

As a result of this, R= $\sum_{t \in C} p^t \sum_{n \in N} x^t_n$ xt n becomes a concave function as $x^t_n = \delta_n(1 - p^t/w_n)$ for all n 2 N. Also, the cost function is concave as composition of two functions that are concave is concave. This makes P, a convex optimization function.

4.0 OPTIMAL SCHEDULING

In Stackelberg game retailer and customer cater to their own profits while in optimal scheduling the only aim is to minimize the generation cost meanwhile satisfy the customers' charging requirements. Hence, there is no need for price signaling in order to enable consumption control.

The optimal scheduling problem is given as:

$$min_x \sum_{t \in C} G(X^t) \quad ...(15)$$

$$\sum_{t \in T} x^t_n = E_n \; for \; all \; n \in N \quad ...(16)$$

$$0 \leq x^t_n \leq \delta_n, \; for \; all \; n \in N \; and \; t \in C \quad ...(17)$$

1) Single-customer case: For the case of single-customer, the objective function can be written as $G(X^t) = a(x^t_o + x^t_i)^2$. Consider the case with zero base loads, then the solution turns out to be:

$$x^{t*}_{i,o} = \frac{E_i}{\mu_c C_i} \; for \; all \; t \in C_i \quad ...(18)$$

As the only aim of this optimization problem is to minimize the cost of generation, the electricity consumption level needs to be made as much flat as possible. For same base load throughout the day, (18) is the consumption value. However, when the base loads differ on an hourly basis, the results also vary.

2) Multiple-customers' case: The objective function in this case is $G(X^t) = a(x^t_o + \sum_{n \in N} x^t_i)^2$, which is a convex optimization problem to minimize the generation cost. For details of the solution method refer to section V. This turns out to be an optimization problem that explodes dimensionally when more EVs are deployed in the system. This calls for special techniques for solving such optimization problems as mentioned in section V.

5.0 SOLUTION TECHNIQUE

The Solution Technique used is PSO which is an evolu- tionary meta-heuristic algorithm. PSO is a continuous domain multivariable search optimizer that is inspired by the behavior of migratory birds. The agents are randomly generated in the search space and they move under the influence of social effect and self-experienced effect along with the inertial motion to go to successive positions as iterations move head. The idea is that the whole population set moves toward the global minimum as a swarm of birds and finally reach the near-optimal solution. This technique was proposed by Kennedy and Eberhart in 1995 and has since been widely used as an optimization tools in may fields.

PSO is a real domain function optimizer. The particles are randomly initialized in the search space and they move every iteration based the social, inertial and self-effects. This is a meta-heuristic approach and can solve any function with constraints like a black box. This section describes how PSO is used in this paper to solve the Stackelberg Game problem. PSO is made to solve the price setting problem for the follower given a speculative equation for consumer consumption that maximises the consumer utility is already known. This will allow the decision variables (prices to be set by retailer in this case) to be set by PSO while meeting the required constraints. The process of PSO solving the Stackelberg problem is shown in the flow chart in Figure 2.

The objective of the PSO would be to optimize and max- imize the profit for the retailer who is the follower in this case. The PSO sets the final prices to maximize the profit on speculative consumption as described in the section III. On the other side PSO can also be used to solve an optimum scheduling of loads instead of setting prices, with the main objective to reduce the total cost of consumption for the retailer. This is the global optimum consumption to minimize the total cost of electricity while satisfying the constraints as mentioned in section IV. The flow chart Figure 3 describing the implementation of PSO for optimal scheduling is shown below:

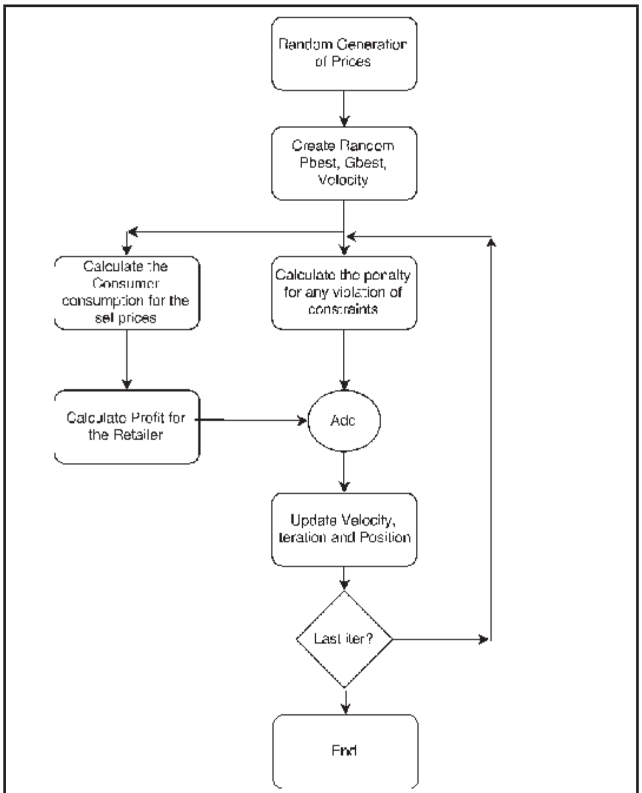

FIG. 2. FLOWCHART OF PSO AS APPLIED TO SOLVE STACKELBERG GAME

case. Figure 4 shows the equilibrium prices obtained for one, two, three and five customers' cases game as set by the UC for ten hours. The ten hours are basically 8pm in the evening to next morning 6:00am. Since, we consider at-home charging of EVs in our analysis, this duration is justifiable as most of the EVs are parked at night and are available for at-home charging. Here, the analysis has been performed on historical data. Table I gives the values of different parameters for different number of customers' cases. For the single customer case, we have taken the following values for the variables in our game: w=10, delta=5.4 and energy requirement as 8.5kWh. Similarly, as the number of customers go on increasing their weights are assumed to be same while the charging rates in different number of customers cases are summed up to give the total charging rate. Energy requirements of number of customers are also summed up in the multiple EV customers' case.

6.0 TEST SYSTEM

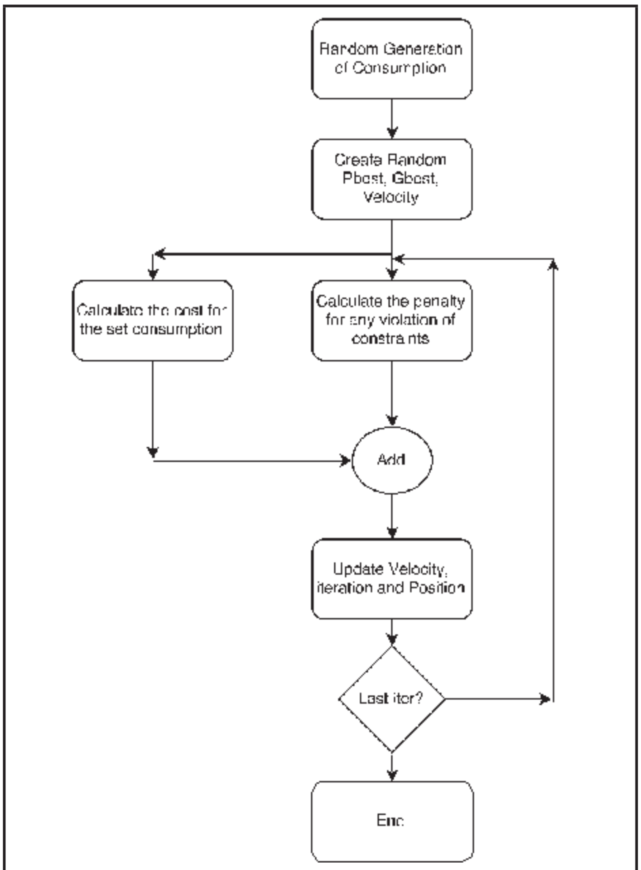

FIG. 3. FLOWCHART OF PSO AS APPLIED TO OBTAIN OPTIMUM SOLUTION

Here we have considered various cases with different num- ber of EV customers participating in DR programs, in order to study the applicability of our proposed game. Beginning with one customer case (the term 'one customer' refers to one EV customer and so on and so forth), we have performed our analysis for two, three and five customers cases. The proposed Stacklberg game has been compared with optimum scheduling

TABLE 1

DATA FOR SINGLE, TWO, THREE AND FOUR CUSTOMERS' CASE

Different parameters	Single customer	Two customers	Three customers	Five customers
w δ(kW) E (kWh)	10 5.4 8.5	10 10.8 17	10 16.2 25.5	10 27 42.5

7.0 RESULTS AND DISCUSSION

On solving the Stackelberg game with PSO as an op- timization tool, we obtained the equilibrium prices. Based on these prices, the consumption was determined for the customer. In order to compare the results of our game with optimum case, we determined consumption using optimum scheduling also, where the only objective is to minimize the cost of consumption. As the objective for optimum case is to minimize the cost, it shows the tendency to flatten the electricity consumption. Both the optimum and Stackelberg scheduling methods were used. In Figure 5, the variation in electricity consumption of one EV customer with time is shown, as the prices change according to Stackelberg game and Optimum scheduling methods.

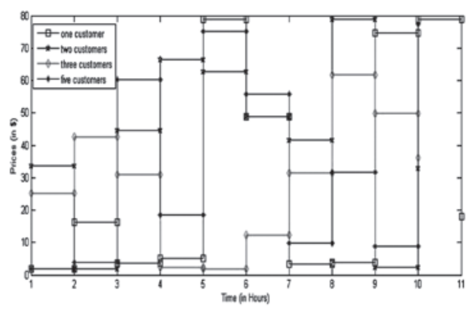

FIG. 4. STACKELBERG GAME PRICES FOR DIFFERENT NUMBER OF EV CUSTOMERS

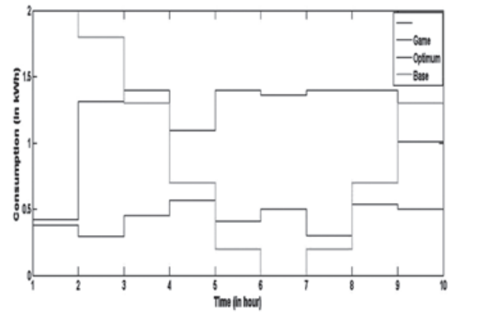

FIG. 5. ELECTRICITY CONSUMPTION OF SINGLE EV CUSTOMER IN STACKELBERG GAME AND OPTIMAL SCHEDULING CASE

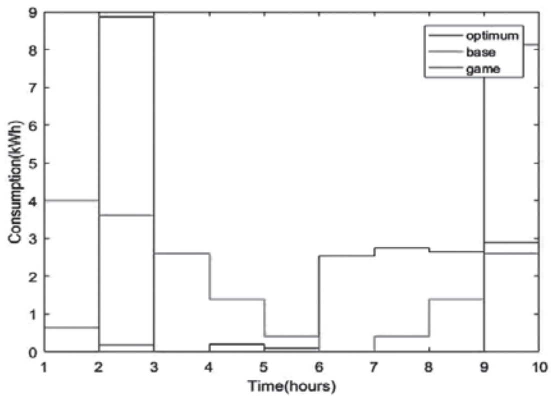

FIG. 6. ELECTRICITY CONSUMPTION OF TWO EV CUSTOMERS IN STACKELBERG GAME AND OPTIMAL SCHEDULING CASE

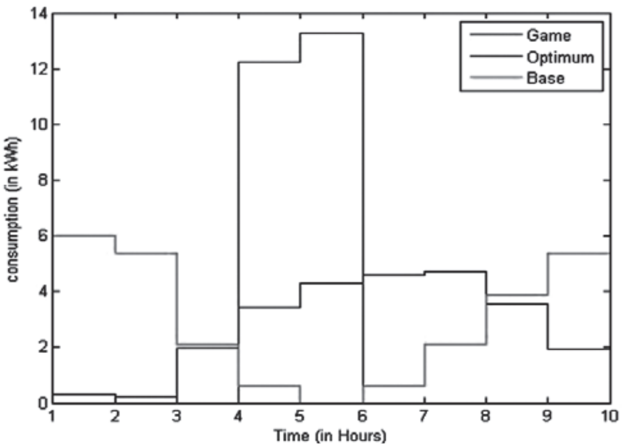

FIG. 7. ELECTRICITY CONSUMPTION OF THREE EV CUSTOMERS IN STACKELBERG GAME AND OPTIMAL SCHEDULING CASE

The Load, Base and Optimal schedules for two customers case is shown in Figure 6. It can be seen that the EVs charge only for two hours according to the prices obtained by Stackelberg game and the consumption according to Optimum case is com- paratively more uniform. While talking about three customers case, Figure 7 shows how the charging occurs majorly during the two hours when the prices obtained from game are very low. The optimum case tends to have a uniform consumption on the other hand. Similarly, Figure 8 shows the consumption for five customers case and the charging of EVs majorly takes place in the beginning of the charging period, according to the game because the prices are low during this time period. While the consumption according to optimum case again tends to be flatter. In case of Stacelkberg game, the customer consumption pattern for any multiple customers case is peaky (high peak to average ratio) as seen from Figure 5, 6, 7 and 8. While the optimal consumption pattern for the customers has a close to unity peak to average ratio.NO

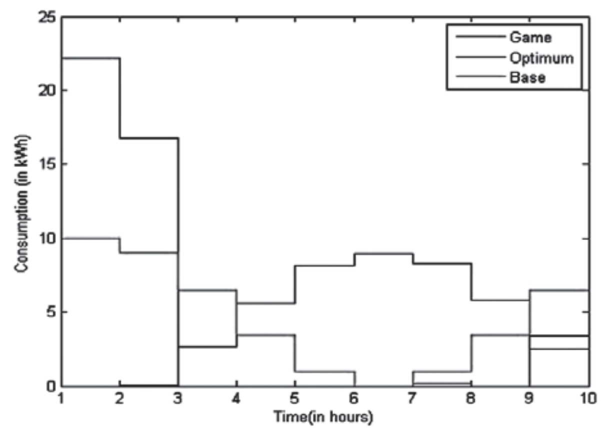

FIG. 8. ELECTRICITY CONSUMPTION OF FIVE EV CUSTOMERS IN STACKELBERG GAME AND OPTIMAL SCHEDULING CASE

8.0 CONCLUSION

With the increasing concern over climate change and global warming, large scale deployment of EVs is a potential solution. To motivate customers to adopt to EVs and to protect the grid from surge in peak demand, we have formulated a Stackelberg game based demand response program, in order to manage the EV charging. This game is essentially designed to address the interaction between a single retailer and multiple EV users meanwhile maximizing the payoffs of all the entities involved. PSO has been used as the tool for optimization. A Stackelberg game for single customer and two customers' case is discussed and compared with that of optimum charging in both the cases. The electricity consumption patterns obtained from Stackelberg game and Optimum charging are different and indicative of the fact that appropriate pricing strategy needs to be adopted.

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