

## Application of mathematical interpolation technique in nano based insulating mineral oil

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*In general, insulating oil serves as an insulation and cooling medium of power transformers and different High Voltage (HV) application. The properties of insulating oil like breakdown voltage (BDV), viscosity, flash point and fire point are enhanced by adding of different additives in an optimal way. In this work, a mathematical model has been developed by using Lagrange interpolation technique for predicting the critical properties values in different volume concentrations (in Vol %). The predicted critical properties are compared with the set of measured data. Based on this approach, accuracy of predicted value has been obtained in between the two available measured data in two known different volume concentrations. From the volume concentrations (0-0.01%), the error seems to be negligible. Hence, Lagrange interpolation technique is considered as a reliable technique for predicting the intermediate value for different concentrations of nano powder doped insulating oil used in different HV power apparatus.*

**Keywords:** *Insulating oil, Critical parameters, Interpolation technique, Nanofluids.*

### 1.0 INTRODUCTION

Life span of power transformer is mainly depending upon the insulation and its cooling system. Since inception of transformers, mineral oil is the preferred choice as insulating medium of transformers. With the increase of demand and growth in power system, rating of transformers are linearly increasing. In order to counter these constraints, properties of insulating medium should be enhanced [1]. In last few decades, for achieving the above goal, a lot of research work has been undertaken in the areas of enhancement of insulating material properties [2]. Enhancement in insulating properties of material is mainly achieved by doping of different nano-additives, anti-oxidants, inhibitors and so on.

Till now many additives such as  $\text{SiO}_2$ ,  $\text{TiO}_2$ ,  $\text{CaCu}_3\text{Ti}_4\text{O}_{12}$ ,  $\text{Fe}_2\text{O}_3$ ,  $\text{Al}_2\text{O}_3$  in nano domain and  $\alpha$ -Tocopherol, Citric acid, Butylated Hydroxyl Anisole etc. in anti-oxidants domain have been used for the enhancement studies [3]-[4] and [5]. It is Understood from the above studies that, these materials are enhancing the properties of oil than the base values. In this work, an attempt has been made for optimum use of additives by developing a mathematical Model. In order to investigate the enhancement properties of nano based insulating oil, nanomaterial with different volume fractions is mixed with insulating oil, an average of four to six volume fractions is considered in account for stable result [6]. Testing of enhancement properties of nano based transformer oil in each individual concentration

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of nano material is practically tedious and time consuming task. Further in addition, the study of above properties in between two concentration limit of additives is an impossible task as infinite number of concentration points is possible for the studies. Hence, in order to find out the above optimum properties in between any two-concentration of additives, different mathematical tools are generally applied so that it can give an optimum additive concentration. In this work, for optimizing the additives volume concentration in insulating oil with better accuracy, a mathematical model has been developed by using Lagrange interpolation technique which is a unique tool for such investigation.

## 2.0 INTERPOLATION

Interpolation technique is used to estimate the intermediate values for given set of data of two points. In this technique, mainly two types of variables are used i.e. dependent and independent variables. Interpolation is a very primitive technique, used in olden times by Babylonian astronomers for linear and higher order interpolation to fill gaps in ephemerides of the sun, moon, and the then known planets. Linear interpolation techniques are considered as "chord function" (related to the sine function) for the purpose of computing the position of celestial bodies. And also, adaptive linear interpolation technique is applied in constructing tables of functions for more than one variable defined for astronomical purposes [7]. Here, Lagrange interpolation technique is used with a mathematical equation where the independent variable is the volume concentration of additives and dependent variable are the four critical parameters such as BDV, Viscosity, flash point and fire point which are explained in details in below.

### 2.1. Lagrange Interpolation

If 'n' no. of independent variables as  $x_1, x_2, x_3, \dots, x_n$  and corresponding function value  $f_1, f_2, f_3, \dots, f_n$  is given then the points  $(x_1, f_1), (x_2, f_2), (x_3, f_3), \dots, (x_n, f_n)$  can be taken as the values connected by a curve. Any function satisfying the condition like

$$P(x_k) = f_k \quad \dots(i)$$

For  $k = 0, 1, 2, \dots, n$

The equation can be represented as

$$P_n(x) = \sum_{i=1}^n f_i I_i(x) \quad \dots(ii)$$

Where

$$I_i(x) = \prod_{(j=0, j \neq i)}^n \frac{(x-x_j)}{(x_i-x_j)} \quad \dots(iii)$$

The above equation is called as Lagrange interpolation polynomial [8]-[9].

## 2.2. Critical Parameters

In this paper four critical parameters have been taken for analysis purpose. The definition and significance of these parameters are as follows-

### 2.2.1. Breakdown voltage

As per the standard ASTM D877 Breakdown voltage is the ability of insulating oil to withstand the voltage stress. Breakdown voltage of insulating oil is dependent on the different parameter such as humidity, acidity, presence of contaminants etc. [10].

### 2.2.2. Viscosity

Viscosity is measured by the timing of the flow of a known volume liquid through a tube. The property of Viscosity have an impact on the movement of electron in insulating oil in electrical equipment. It is also having an impact on the cooling phenomena of oil which is described in the standard ASTM D445 [11].

### 2.2.3. Flash point and Fire point

According to standard ASTM D92-16b, Flash point of mineral oil is the temperature at which the oil makes sufficient vapor to give a flammable mixture with air and fire point is the temperature at which the vapormakes a mixture with air which can give fire at least [12].

## 2.3. Mathematical Modelling

Experimental data for four critical parameters of nano based transformer oil has been collected

from published data [6]. The measurements were undertaken in high voltage and insulation laboratory, National Institute of Technology Durgapur and the measured data are presented in Table 1. Based on the experimental value, interpolation based mathematical equations are formulated in which four independent variable  $x_1, x_2, x_3$  and  $x_4$  are the four concentrations and corresponding dependent variables are  $f_1, f_2, f_3$  and  $f_4$  for each critical parameter as shown in Table 1 and Table 1A.

TABLE 1				
BASE VALUES FOR CRITICAL PARAMETERS				
S.N.	Conc. (%)	BDV value (kV)	Viscosity (C-st)	
			At RTP	At 90°C
1	0	34	13.37	4.1
2	0.005	41.2	13.51	4.13
3	0.01	46.4	13.55	4.18
4	0.05	44.6	13.82	4.21

TABLE 1 A			
BASE VALUES FOR CRITICAL PARAMETERS			
S.N.	Conc. (%)	Flash Point (°C)	Fire Point (°C)
1	0	151	168
2	0.005	157	169
3	0.01	159	174
4	0.05	162	178

Mathematical equation based on Lagrange interpolation techniques for the above four parameters are given as below.

### 2.3.1. AC Breakdown voltage

BDV of nano based transformer oil is an important parameter for diagnosis of health of the transformer and it is most preferable indicator among electrical properties. Therefore, it is necessary to investigate the BDV value at every possible concentration for better analysis of the health of a transformer. The Lagrange interpolation equation for breakdown voltage is given as-

$$P_3(x) = 0.029 \times 10^7 \times x^3 - 0.444 \times 10^5 \times x^2 + 1.644 \times 10^3 \times x + 34 \quad \dots(iv)$$

Where  $0 \leq x \leq 0.005$

### 2.3.2. Viscosity

Viscosity of any insulating liquid gives the effectiveness of liquid as coolant. As the value of Viscosity depends upon the temperature, two equations are presented for the analysis of viscosity at different temperatures, one at RTP and other at a temperature of 90°C.

The Lagrange interpolation equation of Viscosity at RTP is given as-

$$P_3(x) = 0.003 \times 10^7 \times x^3 - 0.032 \times 10^5 \times x^2 + 0.0439 \times 10^3 \times x + 13.37 \quad \dots(v)$$

and at 90° C, it is given as-

$$P_3(x) = 2.6 \times 10^4 \times x^3 - 0.016 \times 10^5 \times x^2 + 0.0128 \times 10^3 \times x + 4.1 \quad \dots(vi)$$

Where  $0 \leq x \leq 0.05$

### 2.3.3. Flash point and Fire point

In transformer, routine measurement of flash and fire point are necessary for maintaining the healthy transformer. High flash and fire point of insulating liquid, means for better insulation of electrical equipment. In order to obtain an optimized value of Flash and Fire point of additive mixed insulating oil, it is necessary to investigate the values at different concentrations.

The Lagrange interpolation equation for Flash point is as follows-

$$P_3(x) = 0.14 \times 10^7 \times x^3 - 1.015 \times 10^5 \times x^2 + 1.665 \times 10^3 \times x + 151 \quad \dots(vii)$$

And for Fire Point is given as-

$$P_3(x) = 0.0203 \times 10^7 \times x^3 - 1.104 \times 10^5 \times x^2 + 0.3012 \times 10^3 \times x + 168 \quad \dots(viii)$$

Where  $0 \leq x \leq 0.05$

Equation no. (iv), (v), (vi), (vii) and (viii) are the Lagrange Interpolation equation for predicting the interpolating values in between the concentrations for critical parameters.

### 3.0 RESULTS AND DISCUSSION

The predicted values by Lagrange interpolation technique for breakdown voltage (BDV), viscosity, flash point and fire point of nano based insulating material are presented in the concerned Tables and Plots. The tables are containing the different concentrations, actual measured values, predicted values and percentage error. The graphs are plotted with interpolated value and actual value for each parameter of additive based insulating oil. In Figures, the blue lines with star points is showing interpolated values at different concentration and red line with points are denoting the plot of actual value. In this work, measured values of four concentration points i.e. 0, 0.005, 0.01 and 0.05 are used for analysis. For interpolation, seven concentration points have been taken i.e. 0, 0.0025, 0.005, 0.0075, 0.01, 0.03 and 0.05. Here experimental value is considered as actual measured value which are used for the analysis. The analysis of the above four parameters of nano based insulating oil are presented in the following sections.

#### 3.1. AC Breakdown voltage

The interpolated values have been calculated for breakdown voltage based on the Lagrange interpolation equation(iv). The different breakdown voltage values with interpolated points and corresponding percentage error are given in Table 2. From Table 2, it is observed that percentage of error till concentration of 0.01% is very less, therefore up to 0.01% concentration of additives, the interpolated values are having less error but for the concentration value between 0.01% to 0.05%, the interpolated values will drastically differ from actual value as the error reaches in 7.02% for additive concentration of 0.05%.

No.	Conc. (%)	Actual Value (kV)	Lagrange Prediction	% Error
1	0	34	34	0
2	0.0025	-	37.83	-
3	0.005	41.2	41.14	0.14
4	0.0075	-	43.95	-
5	0.01	46.4	46.29	0.23
6	0.03	-	51.19	-
7	0.05	44.6	41.45	7.06

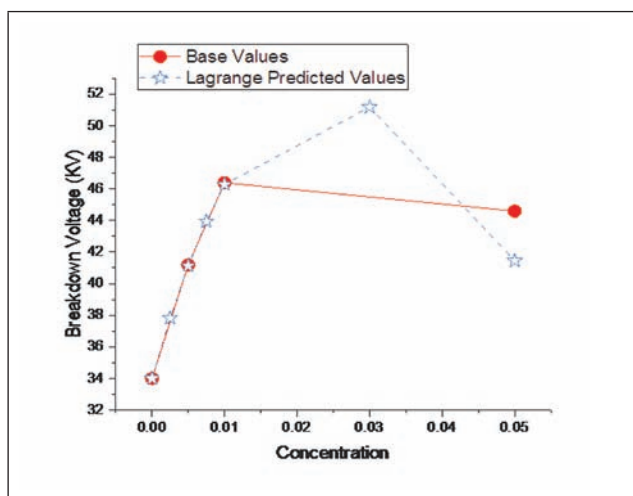


FIG. 1 BREAKDOWN VOLTAGE WITH INTERPOLATED VALUE AND BASE VALUE

In the BDV characteristic, it is seen that breakdown value from concentration of 0% to 0.01%, the interpolated values are very close to the actual value. From Figure 1, it is seen that the actual points are also on the same curve till 0.01% concentration. The actual Breakdown voltage value is reduced for concentration 0.05%. So in between 0.01% and 0.05%, an optimized point has been examined which lies in 0.03% concentration of additive and the breakdown voltage value is calculated as 51.19 kV.

#### 3.2. Viscosity

The mathematical equation for viscosity is given in equation no. (v) and (vi). Based on the equations, the interpolated values have been calculated at RTP and at 90°C. The data points and characteristic of viscosity for RTP is given in Table. 3 and for 90°C in Table.4

**TABLE 3**  
**INTERPOLATED DATA FOR VISCOSITY AT RTP**

S.N.	Conc. (%)	Actual Value (c-St)	Lagrange Prediction	% Error
1	0	13.37	13.37	0
2	0.0025	-	13.46	-
3	0.005	13.51	13.51	0
4	0.0075	-	13.53	-
5	0.01	13.55	13.51	0.22
6	0.03	-	12.61	-
7	0.05	13.82	11.31	18.12

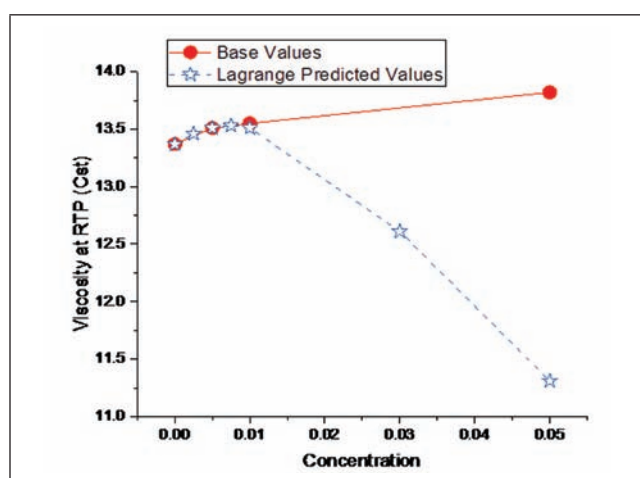


FIG. 2 VISCOSITY PROPERTY AT RTP WITH INTERPOLATED VALUE AND BASE VALUE

The interpolated values for concentration points 0% and 0.005% is approaching exactly same as actual value i.e. the percentage of error is 0%. For 0.01% concentration, the error approaches to 0.22% which may be considered as close to actual value.

**TABLE 4**  
**INTERPOLATED DATA FOR VISCOSITY AT 90°C**

S.N.	Conc. (%)	Actual Value (c-St)	Lagrange Prediction	% Error
1	0	4.1	4.1	0
2	0.0025	-	4.12	-
3	0.005	4.13	4.127	0.07
4	0.0075	-	4.1169	-
5	0.01	4.18	4.094	2.05
6	0.03	-	3.746	-
7	0.05	4.21	3.99	5.22

It is observed from Figure 2 that till concentration of 0.01%, interpolated points and actual points are on the same curve, but beyond 0.01%, the error reaches to 18.12% for additive concentration of 0.05%.

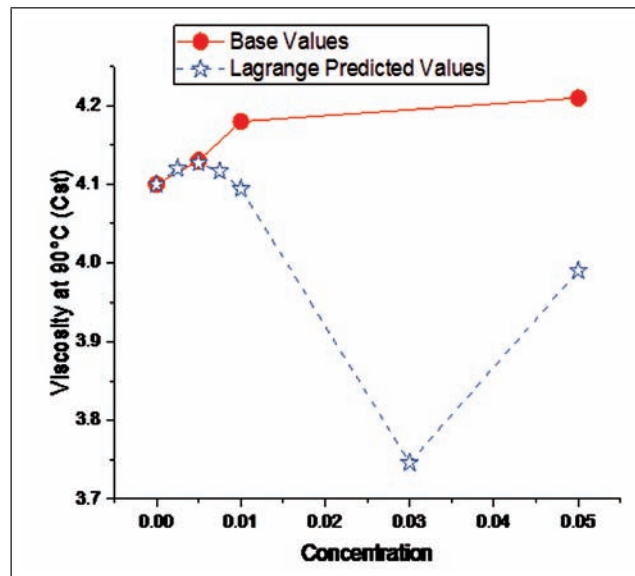


FIG. 3 VISCOSITY PROPERTY AT 90°C WITH INTERPOLATED PREDICTED VALUE AND BASE VALUE

For Viscosity at 90°C, till 0.005% concentration, predicted values are close to actual value, the error is 0.07% but for 0.01% it reaches to 2.05% which is different from the value obtained in RTP. Therefore, it is violating from the properties of previous characteristics. It is also observed that for 0.05% concentration, error reaches to 5.22% which is very less from previous properties compare to 18.12%. In Figure 3 it is observed that till 0.005% concentration, actual and interpolated points are coming on same curve, beyond this, interpolated values are in wide variation i.e. for 0.01% concentration, the error is 2.05% and for 0.05% concentration the error is 5.22%.

### 3.3. Flash Point and Fire Point

The mathematical equations for Flash and Fire Point are given in equations (vii) and (viii). The interpolated values for both parameters are tabulated in Table 5 and Table 6 and the interpolated characteristics are shown in Figure 4 and Figure 5 respectively.



No.	Conc. (%)	Actual Value	Lagrange Prediction	% Error
1	0	151	151	0
2	0.0025	-	154.55	-
3	0.005	157	156.96	0.025
4	0.0075	-	158.36	-
5	0.01	159	158.9	0.062
6	0.03	-	147.4	-
7	0.05	162	155.5	4.01

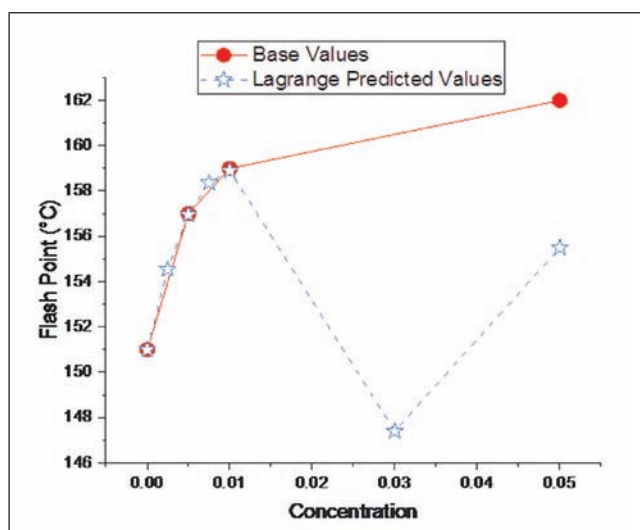


FIG. 4 FLASH POINT PROPERTY WITH PREDICTED VALUE AND BASE VALUE

From the Table. 5 and plot, it is observed that till 0.01% concentration, values are best fit which is nearer to actual value. But beyond this concentration, the value is having a wide variation from its actual value. It is observed that for 0.03% concentration, the value is very less to its nearer values and for 0.05% concentration, the error is 4.01%.

From Table 6, it is observed that the error is very less for all concentrations. For initial two concentration, the error is 0%. For 0.03% concentration a peak value of 203.51°C is obtained, therefore 0.03% concentration of additives may be considered as an optimized value for Fire point.

No.	Conc. (%)	Actual Value	Lagrange Prediction	% Error
1	0	168	168	0
2	0.0025	-	167.90	-
3	0.005	169	169	0
4	0.0075	-	171.09	-
5	0.01	174	173.99	0.005
6	0.03	-	203.51	-
7	0.05	178	175.19	1.57

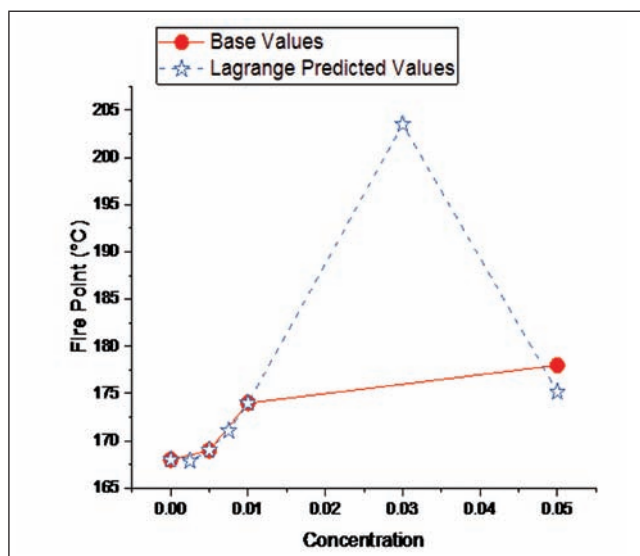


FIG. 5 FIRE POINT PROPERTY WITH PREDICTED VALUE AND BASE VALUE

It is observed from Figure 5 that up to concentration of 0.01% the actual and predicted values are very close, in 0.03% concentration, the values are in wide gap however in 0.05% concentration, again the predicted values are very close to the actual values.

#### 4.0 CONCLUSION

From the overall result, it is concluded that,

- The concentration range between 0% to 0.01% interpolated values are very close to the actual values. Beyond, 0.01% to 0.05% concentration value of function is differing from actual value resulting in more percentage error.

- Beyond the 0.01% concentration the interpolated value is showing more deviation from actual value.

Hence, Lagrange interpolation technique is considered as a reliable technique for predicting the intermediate value for different concentrations of nano powder mixed insulating material which may be also applied for different additives in various insulating medium.

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